



# A2 Level Physics

Chapter 10 – Capacitors

10.1.2 Capacitors

Worked Examples

## Capacitors

### Exam Style Question 1

Fig.1 shows two capacitors, *A* of capacitance  $2\mu F$ , and *B* of capacitance  $4\mu F$ , connected in parallel. Fig. 2 shows them connected in series. A two-way switch *S* can connect the capacitors either to a d.c. supply, of e.m.f.  $6V$ , or to a voltmeter.

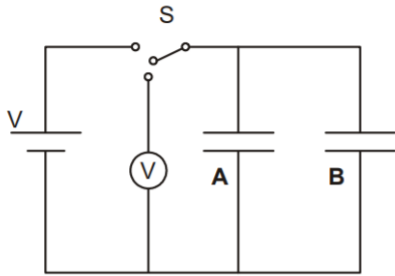


Fig. 1

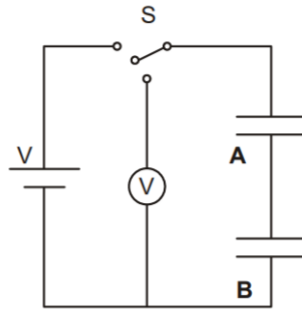


Fig. 2

- (a) Calculate the total capacitance of the capacitors
- (i) when connected as in Fig. 1
- (ii) when connected as in Fig. 2
- (b) The switch in the circuit shown in Fig. 1 is then connected to the battery. Calculate
- (i) the potential difference across capacitor A
- (ii) the total charge stored on the capacitors.
- (c) The switch in the circuit shown in Fig.2 is then connected to the battery. Calculate the total energy stored in the two capacitors.
- (d) The switch *S* in the circuit of Fig. 1 is moved to connect the charged capacitors to the voltmeter. The voltmeter has an internal resistance of  $12 M\Omega$ .
- (i) Explain why the capacitors will discharge, although very slowly.
- (ii) Calculate the time *t* taken for the voltmeter reading to fall to a quarter of its initial reading.

## Capacitors

### Exam Style Question 1

(a) Calculate the total capacitance of the capacitors

(i) when connected as in Fig. 1

For any number (*n*) of capacitor connected in parallel, the total capacitance (*C*) is given by:

$$C = C_1 + C_2 + \dots + C_n$$

$$\therefore C_p = 2 \mu F + 4 \mu F$$

$$C_p = 6 \mu F$$

(ii) when connected as in Fig. 2

For any number (*n*) of capacitor connected in series, the total capacitance (*C*) is given by:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

$$\frac{1}{C} = \frac{1}{2 \mu F} + \frac{1}{4 \mu F}$$

$$\frac{1}{C} = \frac{3}{4}$$

$$\therefore C_s = \frac{4}{3} = 1.33 \mu F$$

(b) Calculate

(i) the potential difference across capacitor A.

Remember the voltage across components in parallel is the same for each component. As the e.m.f. supply is  $6V$  therefore the capacitor A in Fig. 1 has potential difference of  $6V$ .

(ii) the total charge stored on the capacitors.

Use  $C_p = \frac{Q}{V}$  and rearrange for *Q*

$$Q = C_p V = (6 \mu F)(6 V) = 36 \mu C$$

## Capacitors

### Exam Style Question 1

Fig. 1 shows two capacitors,  $A$  of capacitance  $2\mu\text{F}$ , and  $B$  of capacitance  $4\mu\text{F}$ , connected in parallel. Fig. 2 shows them connected in series. A two-way switch  $S$  can connect the capacitors either to a d.c. supply, of e.m.f.  $6\text{ V}$ , or to a voltmeter.

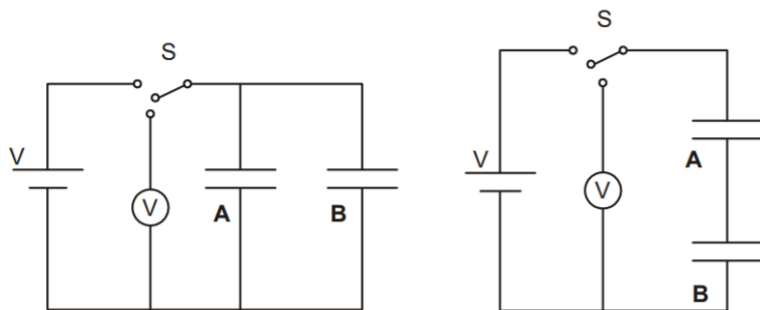


Fig. 1

Fig. 2

- (a) Calculate the total capacitance of the capacitors
- when connected as in Fig. 1
  - when connected as in Fig. 2
- (b) The switch in the circuit shown in Fig. 1 is then connected to the battery. Calculate
- the potential difference across capacitor  $A$
  - the total charge stored on the capacitors.
- (c) The switch in the circuit shown in Fig. 2 is then connected to the battery. Calculate the total energy stored in the two capacitors.
- (d) The switch  $S$  in the circuit of Fig. 1 is moved to connect the charged capacitors to the voltmeter. The voltmeter has an internal resistance of  $12\text{ M}\Omega$ .
- Explain why the capacitors will discharge, although very slowly.
  - Calculate the time  $t$  taken for the voltmeter reading to fall to a quarter of its initial reading.

## Capacitors

### Exam Style Question 1

(c) Calculate the total energy stored in the two capacitors.

$$\text{Use } E = \frac{1}{2} C_S V^2$$

$$E = \left(\frac{1}{2}\right) (1.33 \times 10^{-6} \text{ F})(6 \text{ V})^2$$

$$E = 2.4 \times 10^{-6} = 2.4 \mu\text{J}$$

(d) The switch  $S$  in the circuit of Fig. 1 is moved to connect the charged capacitors to the voltmeter. The voltmeter has an internal resistance of  $12\text{ M}\Omega$ .

(i) Explain why the capacitors will discharge, although very slowly. The capacitors discharge through the voltmeter.

(ii) Calculate the time  $t$  taken for the voltmeter reading to fall to a quarter of its initial reading.

$$\text{Use } V = V_0 e^{-\frac{t}{CR}}$$

$$\frac{1}{4} V_0 = V_0 e^{-\frac{t}{(6 \times 10^{-6} \text{ F})(12 \times 10^6 \Omega)}}$$

Cancel  $V_0$  as they are on both sides

$$\frac{1}{4} = e^{-\frac{t}{(6 \times 10^{-6} \text{ F})(12 \times 10^6 \Omega)}}$$

Take **natural logarithm on both sides**:

$$\ln\left(\frac{1}{4}\right) = \ln\left(e^{-\frac{t}{(6 \times 10^{-6} \text{ F})(12 \times 10^6 \Omega)}}$$

By laws of logarithms  $\ln(e^A) = A$ , so:

$$\ln\left(\frac{1}{4}\right) = -\frac{t}{72}$$

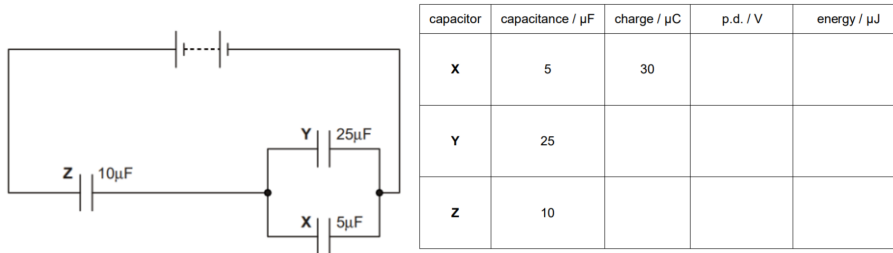
$$t = (72) \left(-\ln\left(\frac{1}{4}\right)\right) = 99.8 \approx 100 \text{ seconds}$$

## Capacitors

### Exam Style Question 2

The charge stored in the capacitor  $X$  of capacitance  $5 \mu\text{F}$  in the circuit given in the figure below is  $30 \mu\text{C}$ .

(a) (i) Complete the table for this circuit.



(ii) Using data from the table find

- the e.m.f. of the battery,
- the total charge supplied from the battery,
- the total circuit capacitance,
- the total energy stored in all the capacitors.

(b) (i) What law or principle of physics was used to determine (a)(ii)1?

(ii) What law or principle of physics was used to determine (a)(ii)2?

(c) The battery is removed and replaced by a resistor of resistance  $200 \text{ k}\Omega$ . The capacitors now discharge through this resistor. Calculate

(i) the time constant of the circuit

(ii) the fraction of the total charge remaining on the capacitors after a time equal to four time constants.

## Capacitors

### Exam Style Question 2

(a)(i) Complete the table for this circuit.

capacitor	capacitance / $\mu\text{F}$	charge / $\mu\text{C}$	p.d. / $\text{V}$	energy / $\mu\text{J}$
X	5	30	$V = Q/C$ $= 30/5$ $= 6 \text{ V}$	$E = \frac{1}{2} CV^2$ $= \left(\frac{1}{2}\right) (5)(6)^2$ $= 90 \mu\text{J}$
Y	25	$Q = CV$ $= 25 \times 6$ $= 150 \mu\text{C}$	Parallel to X therefore: $6 \text{ V}$	$E = 450 \mu\text{J}$
Z	10	$Q = 30 + 150$ $Q = 180 \mu\text{C}$	$V = Q/C$ $= 180/10$ $= 18 \text{ V}$	$E = 1620 \mu\text{J}$

(ii) Using data from the table find

1) the e.m.f. of the battery,

$$e.m.f. = 18 \text{ V} + 6 \text{ V} = 24 \text{ V}$$

2) the total charge supplied from the battery,

Charge on capacitor Z is  $180 \mu\text{C}$  and according to Kirchhoff's first law the total charge supplied from the battery must be  $180 \mu\text{C}$  as capacitor is in series with the battery.

$$Q_T = 180 \mu\text{C}$$

3) the total circuit capacitance,

$$\text{Use } C = \frac{Q}{V}$$

$$C = \frac{180 \times 10^{-6} \text{ C}}{24 \text{ V}} = 7.5 \times 10^{-6} \text{ F} = 7.5 \mu\text{F}$$

4) the total energy stored in all the capacitors.

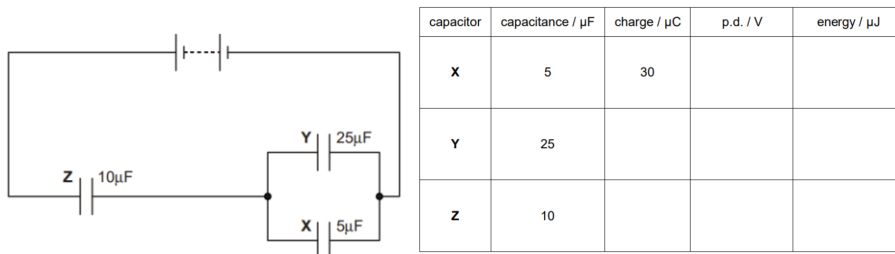
$$E_T = 90 \mu\text{J} + 450 \mu\text{J} + 1620 \mu\text{J} = 2160 \mu\text{J}$$

## Capacitors

### Exam Style Question 2

The charge stored in the capacitor  $X$  of capacitance  $5 \mu\text{F}$  in the circuit given in the figure below is  $30 \mu\text{C}$ .

(a) (i) Complete the table for this circuit.



(ii) Using data from the table find

- 1) the e.m.f. of the battery,
- 2) the total charge supplied from the battery,
- 3) the total circuit capacitance,
- 4) the total energy stored in all the capacitors.

(b) (i) What law or principle of physics was used to determine (a)(ii)1?

(ii) What law or principle of physics was used to determine (a)(ii)2?

(c) The battery is removed and replaced by a resistor of resistance  $200 \text{ k}\Omega$ . The capacitors now discharge through this resistor. Calculate

(i) the time constant of the circuit

(ii) the fraction of the total charge remaining on the capacitors after a time equal to four time constants.



## Capacitors

### Exam Style Question 2

**(b) (i) What law or principle of physics was used to determine (a)(ii)1?**

Kirchhoff's second law.

**(ii) What law or principle of physics was used to determine (a)(ii)2?**

Kirchhoff's first law.

**(c) Calculate**

**(i) the time constant of the circuit,**

Use  $\tau = CR$

$$\tau = (7.5 \times 10^{-7} \text{ F})(200,000 \Omega)$$

$$\tau = 1.5 \text{ seconds}$$

**(ii) the fraction of the total charge remaining on the capacitors after a time equal to four time constants.**

Use  $Q = Q_0 e^{-\frac{t}{CR}}$

$$Q = Q_0 e^{-\frac{4CR}{CR}}$$

$$\frac{Q}{Q_0} = e^{-4} = 0.0183$$

## Capacitors

### Exam Style Question 3

- (a) Define the time constant of a capacitor-resistor discharge circuit.
- (b) A student designs a circuit with a time constant of 5.0 s. State suitable values for resistance  $R$  and capacitance  $C$  for this circuit.
- (c) Fig. 4.1 shows a circuit with a capacitor of capacitance 0.010 F.

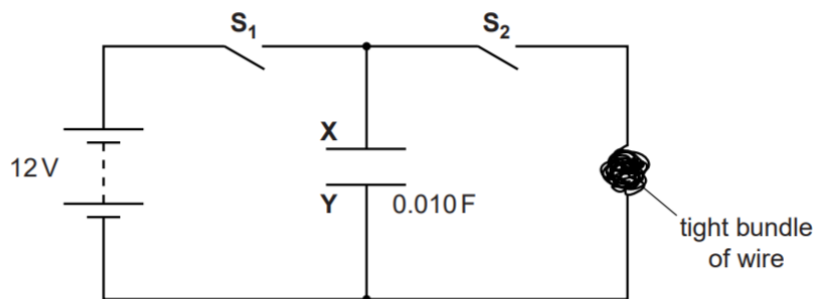


Fig. 4.1

A tight bundle of wire is made from 5.0 m of insulated wire of diameter 0.12 mm and resistivity  $4.9 \times 10^{-7} \Omega m$ . The material of the wire has density  $8900 \text{ kg m}^{-3}$  and specific heat capacity  $420 \text{ J kg}^{-1} \text{ K}^{-1}$ .

- (i) Calculate the time constant of the circuit.
- (ii) Switch  $S_2$  is open. Switch  $S_1$  is closed. Explain in terms of the movement of electrons how X and Y acquire equal but opposite charge.

## Capacitors

### Exam Style Question 3

- (a) Define the time constant of a capacitor-resistor discharge circuit.  
The time taken for the p.d./current/charge to decrease to  $\frac{1}{e}$  of its initial value.

- (b) A student designs a circuit with a time constant of 5.0 s. State suitable values for resistance  $R$  and capacitance  $C$  for this circuit.

We know that  $\tau = CR$  so choose any two easy values such as:

$$R = 5 \text{ M}\Omega$$

$$C = 1 \mu\text{F}$$

When you do enough of these questions you realise that  $R$  are usually in  $\text{M}\Omega$  and  $C$  are usually in  $\mu\text{F}$  so that's why I've picked those units. The other reason is 1 farad is a very large capacitance value and practical capacitors have much smaller capacitances. For this reason smaller sub-multiples are used.

- (c) (i) Calculate the time constant of the circuit.

Recall  $R = \frac{\rho L}{A}$  where  $\rho = \text{resistivity}$ ,  $L = \text{length}$  and  $A = \text{cross sectional area} = \text{area of a circle} = \pi r^2$ .

$$R = \frac{(4.9 \times 10^{-7} \Omega m)(5.0 \text{ m})}{\pi(0.06 \times 10^{-3} \text{ m})^2}$$
$$R = 216.6275 \dots \Omega$$

Now use  $\tau = CR$  to calculate time constant

$$\tau = (0.010 \text{ F})(216.62 \dots \Omega)$$

$$\tau = 2.166 \dots = 2.2 \text{ seconds}$$

- (c) (ii) Explain in terms of the movement of electrons how X and Y acquire equal but opposite charge.

Electrons are removed from X and deposited on Y.

X becomes positive and Y becomes negative.

The size of the charge is the same because an equal number of electrons are removed and deposited on the plates.

## Capacitors

### Exam Style Question 4

(a) Capacitance is measured in farads. Define the farad.

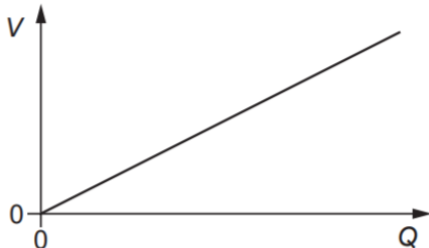


Fig. 1.1

(b) Fig. 1.1 shows the graph of potential difference  $V$  against charge  $Q$  stored for a capacitor of capacitance  $C$ .

State the quantity represented by the

(i) gradient of the graph

(ii) area under the graph.

(c) You are given three capacitors of capacitances  $100\ \mu\text{F}$ ,  $200\ \mu\text{F}$  and  $500\ \mu\text{F}$ . Calculate the minimum total capacitance of these three capacitors in a combination. Show how the capacitors are connected.

(d) A  $0.10\ \text{F}$  capacitor is charged at a constant rate with a steady current of  $40\ \text{mA}$  for a time of  $60\ \text{s}$ . Calculate the final

(i) charge stored by the capacitor

(ii) energy stored by the capacitor.

## Capacitors

### Exam Style Question 4

(a) Capacitance is measured in farads. Define the farad.

$$1\ \text{farad (F)} = 1\ \text{coulomb per volt (CV}^{-1}\text{)}$$

(b) State the quantity represented by the

(i) gradient of the graph

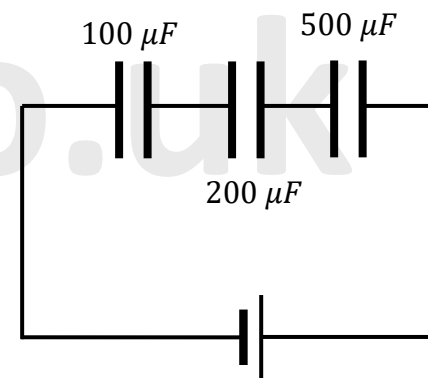
$1/C$  or inverse of capacitance.

(ii) area under the graph.

Work done or energy.

(c) Show how the capacitors are connected.

$$\begin{aligned}\frac{1}{C} &= \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n} \\ \frac{1}{C} &= \frac{1}{100\ \mu\text{F}} + \frac{1}{200\ \mu\text{F}} + \frac{1}{500\ \mu\text{F}} \\ \frac{1}{C} &= 0.017 \\ \therefore C_s &= \frac{1}{0.017} = 59\ \mu\text{F}\end{aligned}$$



(d) Calculate the final

(i) charge stored by the capacitor

Use  $Q = It$

$$Q = 0.040\ \text{A} \times 60\ \text{s} = 2.4\ \text{C}$$

(ii) energy stored by the capacitor.

Use  $E = \frac{1}{2} \frac{Q^2}{C}$

$$E = \left(\frac{1}{2}\right) \left(\frac{(2.4\ \text{C})^2}{(0.10\ \text{F})}\right) = 28.8 = 29\ \text{J}$$

Please see '**10.1.1 Capacitors notes**' pack for revision notes.

For more revision notes, tutorials and worked examples please visit [www.tutorpacks.co.uk](http://www.tutorpacks.co.uk).

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