A2 Level Physics

Chapter 20 – Nuclear and Particle Physics

20.4.1 Nuclear Fission and Fusion

Notes

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Mass Defect

The mass of a nucleus is less than the mass of its individual nucleons – the difference between the masses is called the mass defect (Δm).

So mass defect is defined as:

The mass defect (Δm) of a nucleus is the difference between the total mass of all its separate nucleons and the mass of the nucleus itself.

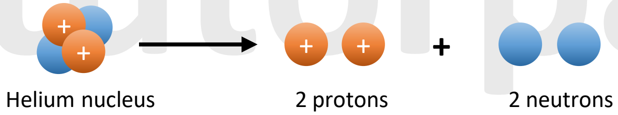
$$\Delta m = m(separate\ nucleons) - m(nucleus)$$

Since nuclear masses are so small, they are measured in atomic mass units (u), rather than kilograms, where:

$$1\ u = 1.6605 \times 10^{-27}\ kg$$

Worked example:

The helium nucleus consists of 2 protons + 2 neutrons.



m(2p + 2n) = $(2 \times 1.00728)u + (2 \times 1.00867)u = 4.0319u$

m(He nucleus) = $4.002602\ u$.

Mass defect, Δm = m(separate nucleons) – m(nucleus)

$$\therefore\ \ \Delta m = 4.0319\ u - 4.002602\ u = 0.029298u$$
$$= 0.029298 \times 1.6605 \times 10^{-27}$$
$$= 4.86 \times 10^{-29} kg\ (2\ d.p.)$$

Mass Defect

Worked example

The mass of a nucleus of iron, ${}^{56}_{26}Fe$, is $55.85\ u$. The mass of a proton is $1.00728\ u$ and the mass of neutron is $1.00867\ u$. Calculate the mass defect of the nucleus in u.

Number of protons is 26, number of neutrons = $56 - 26 = 30$

Mass of nucleons = $(26 \times 1.00728\ u) + (30 \times 1.00867\ u) = 56.44938\ u$

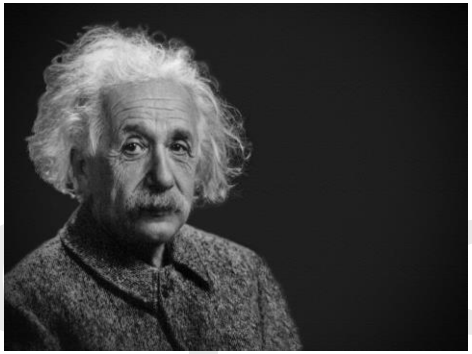
Therefore mass defect = mass of nucleons – mass of nucleus

$$mass\ defect = 56.44938\ u - 55.85\ u = 0.59938\ u$$

Einstein's Equation

When a nucleus is split into its individual nucleons, there is an increase in mass, as we saw from the previous examples. How is that possible, and where does this additional mass come from?

The answer is that as nucleons fuse together, their overall mass reduces, and this 'lost' mass is converted into energy and released. This also works the other way around; the energy required to separate the nucleons from the strong nuclear forces that bind them together is converted into mass.



Albert Einstein proposed that mass and energy are equivalent implying that they are interchangeable quantities.

He mathematically expressed this idea in the following well-known equation:

$$\Delta \boldsymbol{E} = \Delta \boldsymbol{m}\boldsymbol{c}^2$$

Where:

ΔE = energy change in J

Δm = mass change in kg

c = speed of light in a vacuum $(3.0 \times 10^8 ms^{-1})$

In Einstein's equation, mass is measured in kilograms (kg) and energy is measured in Joules (J), however when working with nuclei these units are far too large, thus we are more likely to use the atomic mass unit (u) and the electron-volt (eV) instead.

Binding Energy

Consider disassembling a nucleus by pulling each nucleon apart from its neighbour. To do so, work must be done against the strong nuclear force that holds the nucleons together, resulting in an increase in the potential energy for each separated nucleon.

The Binding Energy of a nucleus is defined as the work that must be done to separate the nucleus into its constituent protons and neutrons.

Binding energy is generally measured in MeV (mega electron-volts).

Binding energy can also be thought of as a measure of how strongly a nucleus is held together.

When protons and neutrons come together to create a nucleus, the strong nuclear force pulls the nucleons together. As a result, an amount of energy equivalent to the nucleus' binding energy is released.

The overall binding energy of a nucleus provides some insight into the nucleus' stability. The higher the nucleus' binding energy per nucleon, the more stable it is.

A better indicator of nuclear stability is the binding energy per nucleon. Bind energy per nucleon is also useful when comparing the binding energies of different nuclei.

The binding energy per nucleon is the average energy required to remove each nucleon from a nucleus, and can be calculated using the formula below:

$$Average\ BE\ per\ nucleon = \frac{Total\ Binding\ Energy\ (B)}{Nucleon\ number\ (A)}$$

Binding Energy

Worked Example

Calculate the binding energy in Mev of the helium-4 nucleus, ${}_{2}^{4}He$, given that its mass defect is $0.0293\ u$. Then calculate the binding energy per nucleon for Helium.

Convert the mass defect into kg:

$$Mass\ defect(\Delta m) = 0.0293 \times (1.661 \times 10^{-27}) = 4.86673 \times 10^{-29} kg$$

Use $E = mc^2$ to calculate the binding energy:

$$E = (4.86673 \times 10^{-29})(3.0 \times 10^{8})^2 = 4.380057 \times 10^{-12} J$$

Now convert to MeV:

Remember to convert from J to eV, you divide by the magnitude of the charge on an electron, $e = 1.60 \times 10^{-19} C$. Then to convert to MeV you just divide by 10^6.

Therefore: Convert joules to eV:

$$E = \frac{4.380057 \times 10^{-12}}{1.60 \times 10^{-19}} = 27375356.25\ eV$$

Then convert from eV to MeV:

$$E = 27375356.25 \div 10^6 = 27.3753 \ldots = 27.4 MeV$$

Therefore the binding energy of Helium-4 is 27.4 MeV.

Binding energy per nucleon for Helium-4 is:

$$BE\ per\ nucleon = \frac{BE}{A} = \frac{27.4}{4} = 6.84\ MeV$$

Binding Energy

The binding energy per unit of mass defect is the same for all nuclei:

$$1\ u \approx 931.5\ MeV$$

Lets revisit the same question again but using the mass defect given above.

Worked example:

Calculate the binding energy in Mev of the nucleus of a helium-4 atom, ${}_{2}^{4}He$, given that its mass defect is $0.0293\ u$.

$$Mass\ defect = 0.0293\ u$$

Therefore binding energy in MeV of helium-4 is:

$$E \approx 0.0293\ u \times 931.5\ MeV = 27.29295 = 27.3\ MeV (3s.f.)$$

Binding Energy

We can prove that $1\ u = 931.5\ MeV$, see below:

We'll need to know some key numbers, such as:

$$1\ u = 1.6605 \times 10^{-27} kg$$
$$c = 2.9979 \times 10^{8}\ ms^{-1}$$
$$1eV = 1.6022 \times 10^{-19} J$$

Now using:

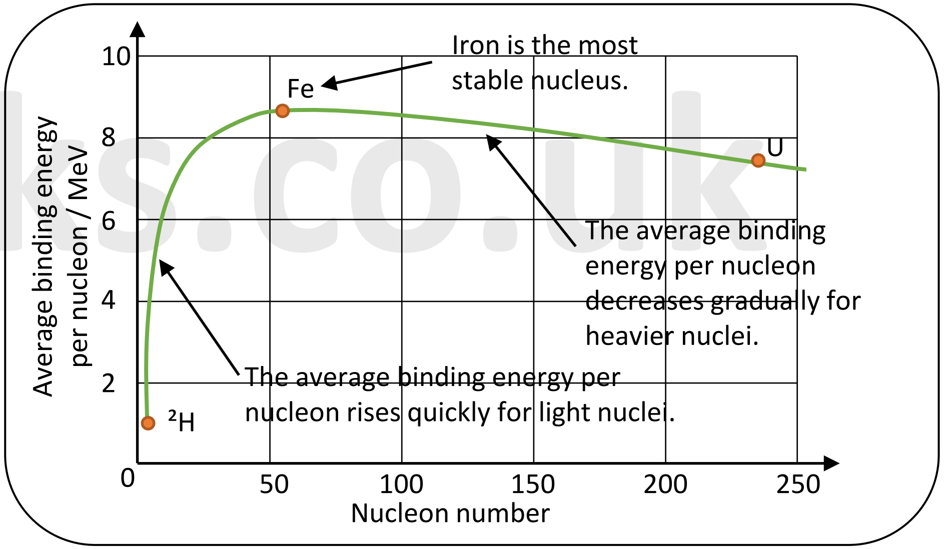
$$E = mc^2 = (1.6605 \times 10^{-27})(2.9979 \times 10^{8})^2 = 1.4924 \times 10^{-10} J$$
$$E = \frac{1.4924 \times 10^{-10} J}{1.6022 \times 10^{-19}\ J\ eV^{-1}} = 931469229.8 = 931.5 \times 10^{6} eV$$
$$= 931.5\ MeV$$
$$1\ u = 931.5\ MeV$$

Binding Energy

For all elements, a graph of average binding energy per nucleon against nucleon number gives a curve similar to the one below. The higher the average binding energy per nucleon means more energy is required to remove nucleons from the nucleus.

As a result, the more stable a nucleus is, the higher its average binding energy per nucleon. This suggests that the most stable nuclei are found **near the graph's maximum point, which is at nucleon number 56**.

With a binding energy per nucleon of roughly **8.79 MeV**, **iron-56** is the **most stable nucleus**.



A graph shows the relationship between nucleon number and the average binding energy per nucleon. The line of best fit, is shown by the green line.

Make sure you understand this graph, including the units and axes, because you may be required to draw it in the exam.

Nuclear Fission

Radioactive decay isn't the only way that nuclei can change – nuclei can split into two smaller nuclei or combine with other nuclei to form larger ones

Large nuclei (such as uranium) are unstable, and some can split into two smaller nuclei at random. This process is known as nuclear fission. The process is referred to as spontaneous if it occurs naturally, or induced if it occurs because of our efforts.

Induced Nuclear Fission

When a slow-moving neutron collides with a U-235 nucleus, induced nuclear fission occurs. The neutron is absorbed, producing a very unstable nucleus (U-236) that splits into two smaller, more stable nuclei (such as barium (Ba-141) and krypton (Kr-92). Several neutrons (usually three) are also emitted, which may induce further fission events. This is referred to as a chain reaction (as seen below), and because each fission event lasts only a fraction of a second, a massive amount of energy can be produced in a very short period of time.

Nuclear Fission

The equation for this fission reaction can be written as follows:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n + energy\ released$$

In a fission reaction there is a decrease in the mass and $\Delta E = \Delta mc^2$ states that mass is converted into energy.

The mass of the U-235 nucleus plus the absorbed neutron is larger than the mass of the fission fragments plus the released fission neutrons. It is this mass difference that causes energy to be released, which appears as KE of the fragments and neutrons. The change in total binding energy equals the energy released.

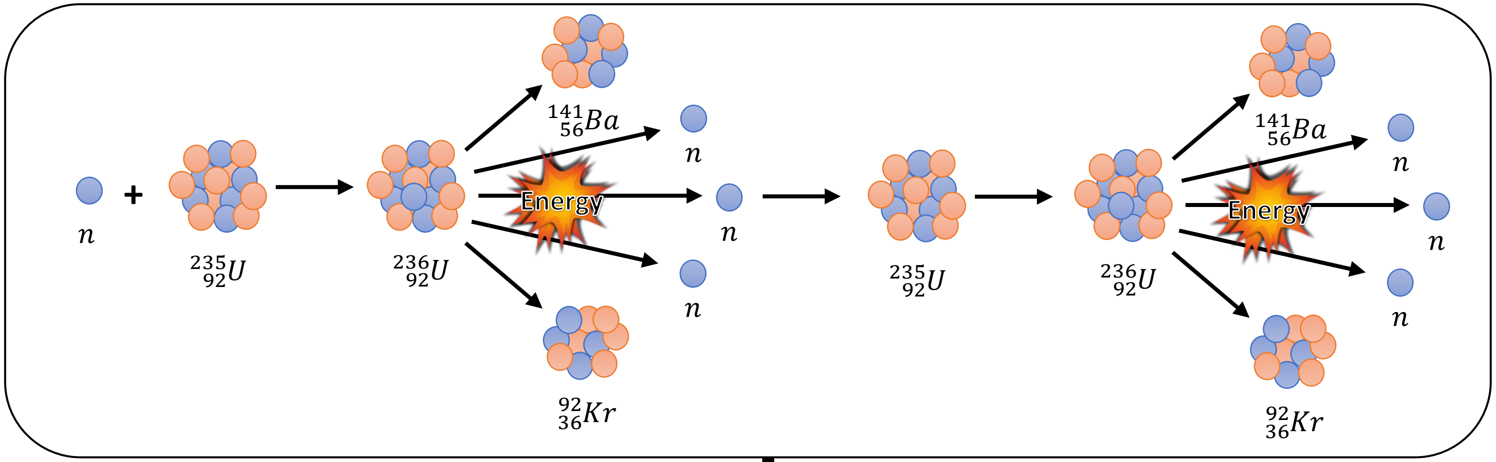
The energy released (ΔE) can be calculated using $\Delta E = \Delta mc^2$, where Δm is the difference between the total mass before and after the fission.

ΔE will be in Joule (J), if the masses are in kg.

If the masses are in u, ΔE can be calculated using:

$$\Delta E = \Delta m \times 931.5\ MeV$$

Which will give you the answer in MeV.



Nuclear Fission Reactor

General Introduction

The energy created by fission, in a nuclear reactor, is recovered and used to make steam, which drives turbines that run generators to produce electricity.

For fission reactions, nuclear reactors use uranium rods rich in U-235 (or occasionally plutonium rods rich in Pu-239) as the 'fuel'.

These fission reactions produce more neutrons, which cause more nuclei to fission in a chain reaction. Thermal neutrons are slowed neutrons that create a chain reaction by allowing them to be absorbed by uranium nuclei.

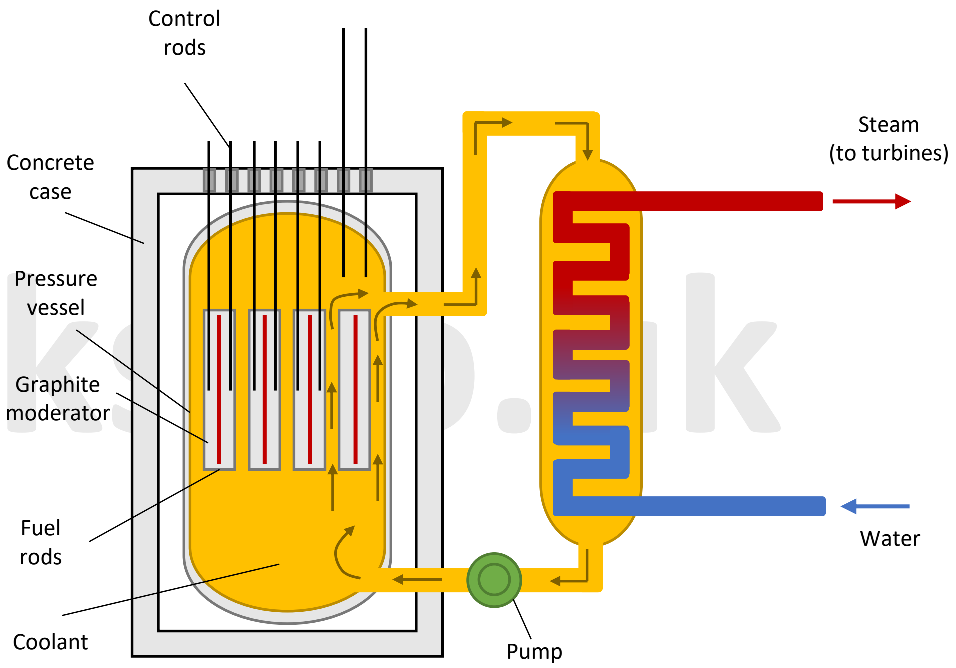
The chain reaction must be carefully controlled in order to extract the energy from fission.

A critical chain reaction is one in which the rate at which neutrons are produced by fissions = the rate at which neutrons are lost.

The chain reaction is said to be supercritical if the rate of fission neutron production exceeds the rate of neutron loss (this is what happens in nuclear fission bombs).

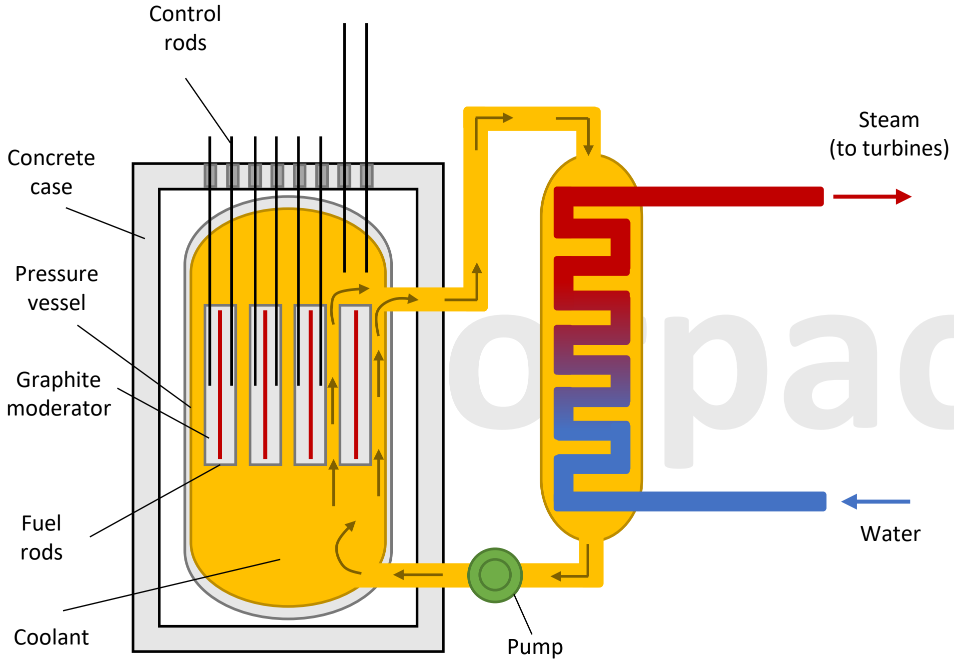
The chain reaction will die out if the rate of fission neutron production is less than the rate of neutron loss.

Nuclear Fission Reactor



The key features of a thermal nuclear reactor

Nuclear Fission Reactor



The key features of a thermal nuclear reactor

Nuclear Fission Reactor

Basic components of a fission reactor

- **Fuel Rods**

Fuel roads contain pallets of fissile material (e.g uranium or plutonium).

The fuel is stored in cylindrical stainless steel rods which are inserted into tubes in the reactor core. When the fuel in the rod is used up, it can be simply removed and replaced.

- **Coolant**

The thermal energy (heat) produced by the fission reactions in the reactor core is removed by circulating a coolant of a gas or liquid (e.g. CO_2 or H_2O) at room temperature around the reactor. The heat is used to heat water and create high-pressure steam, which is used to power the turbines of the generator.

- **Control rods**

The neutrons are absorbed by control rods made of boron. The control rods can be lowered to control the rate of fission and slow down the fission reactions.

- **Moderator**

Moderators (such as water or carbon) surround the nuclear fuel rods to slow down the fasting moving neutrons produced in the fission reactions. The fast neutrons collide elastically (KE is conserved) with the nuclei of the moderator material, slowing them down. The neutrons' successive elastic collisions with the nuclei of the moderator material allow energy to be transferred to the material, slowing down the neutrons.

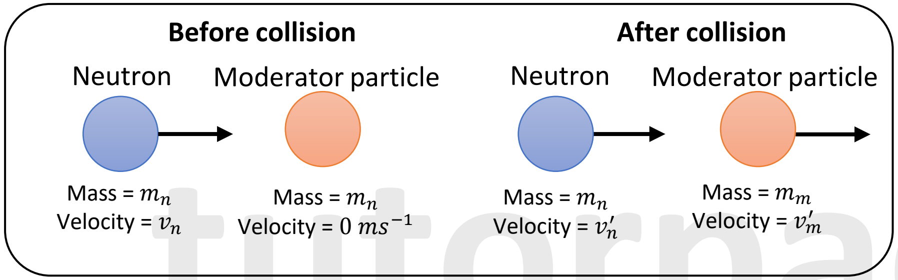
You'll need to pick a moderator that will slow some neutrons down enough to allow them to trigger more fission, keeping the reaction going at a steady pace. Having a moderator with a mass similar to neutrons (such as water) is more effective at slowing down neutrons.

Nuclear Fission Reactor (AQA Only)

Moderation by Elastic Collisions

Fast moving neutrons make elastic collisions with nuclei of the moderator material (such as water or carbon), slowing them down.

Both the KE and momentum are conserved assuming that the collision of a moderator particle and a neutron is perfectly elastic. If we also assume that the moderator particle is stationary before the collision (see below), the equations for the conservation of KE and momentum of the particles can be written as follows.



A diagram showing the velocity and mass of a moderator particle and a neutron before and after an elastic collision.

Conservation of momentum: $m_n v_n = m_n v_n' + m_m v_m'$

Conservation of kinetic energy: $\frac{1}{2}m_n v_n^2 = \frac{1}{2}m_n(v_n')^2 + \frac{1}{2}m_m(v_m')^2$

By using these formulas, it Is possible to get equations for v_n' and v_m' in terms of v_n.

$$v_n' = \frac{(m_n - m_m)}{(m_n + m_m)} v_n \qquad v_m' = \frac{2m_n}{(m_n + m_m)} v_n$$

You don't need to be concerned with how the above equations were rearranged to obtain the v_n' and $v_m{}'$ equations.

Assuming the moderator particle and the neutron have approx. equal masses, that is, $m_n = m_m = m$, the neutrons final velocity would be $0\ ms^{-1}$:

$$v_n' = \frac{(m-m)}{(m+m)} v_n = 0 \qquad v_m' = \frac{2m}{(m+m)} v_n \Rightarrow v_m' = v_n$$

Nuclear Fission Reactor (AQA Only)

Moderation by Elastic Collisions

All of the KE and momentum would be transferred to the moderator particle.

The more similar the mass of a moderator is to the neutron, the more KE and momentum will be transferred from the neutron to the moderator particle.

In a thermal nuclear reactor, you don't want to stop the neutrons, but you do want to slow them down significantly, thus the moderator particles must have a mass that is roughly equal to that of a neutron.

Nuclear Fission Reactor

Safety and Shielding

- **Reactor Shielding**

The reactor's core is contained in a steel pressure vessel that holds the highly pressurised coolant.

The nuclear reactor is encased in a massive concrete casing (about 5 metres thick) that acts as a shield. This keeps radiation from exiting the power plant and reaching the people who work there.

The entire system is enclosed in a steel and concrete containment structure that is meant to keep any radiation from escaping, even in the most extreme case of core meltdown.

- **Emergency Shut-Down**

Control rods can be released into the reactor to automatically shut it down in an emergency. The control rods are fully lowered into the reactor, causing the reaction to slow down as quickly as possible.

Nuclear Fission Reactor

Environmental Effects of Nuclear Waste

Unused uranium fuel rods emit only weakly penetrating alpha radiation, which can be easily contained.

When the uranium fuel rods are used up and fission waste products are created, which typically contain more neutrons than nuclei with similar atomic numbers, making them unstable and radioactive. The fission waste products emit beta and gamma radiation, which are strongly penetrating.

The products can be used in practical applications as medical diagnosis tracers.

Since these products are highly radioactive, they must be handled and stored with extreme caution.

The material is extremely hot when it is withdrawn from the reactor, so it has to be placed in cooling ponds remotely until the temperature drops to a safe level. The radioactive waste should then be enclosed inside glass blocks and placed in thick steel containers encased in concrete before being buried underground until its activity has decreased significantly.

Even though this nuclear waste packing is both safe and effective, the difficulty is that the radioisotopes contained in this packaging have long half-lives and stay harmful for thousands of years, and we don't know how long the packaging will remain intact.

Therefore, these spent fuel rods are classified as high-level radioactive waste.

In addition, there is also intermediate-level radioactive waste, which includes empty fuel rods, contaminated reactor components, and chemical sludges used in the treatment of nuclear fuel. Unlike spent fuel rods, this sort of nuclear waste has low radioactivity. So, this waste is encased in cement within stainless steel barrels and then buried underground in concrete vaults.

Packaging, laboratory tools, and protective clothes are examples of low-level radioactive waste. It is disposed of by compacting and storing it in steel containers, and either burying it or dumping it at sea.

Liquid waste, such as cooling pond water, are cleaned and then released into the sea.

Nuclear Fission Reactor

Benefits and Risks

Nuclear power is used to generate electricity in several countries, including the UK. Some benefits include:

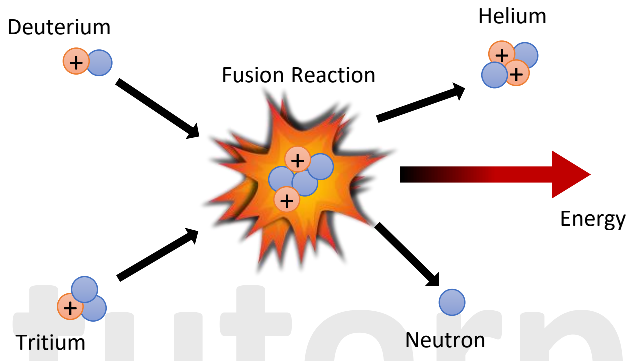
- Nuclear power is efficient; per kg of nuclear fuel, it generates thousands of times more electrical energy than fossil fuels.
- Unlike burning fossil fuels such as coal, oil and gas, there is enough fuel to continue generating electricity using nuclear power for centuries.
- The process also does not produce greenhouse gases, which are harmful to our environment. However, some parts of the process, such as transferring uranium fuel rods to a power plant, do emit greenhouse gases.
- There are, however, risks:
- Nuclear reactors must be designed and built with extreme caution in order to reduce the risk of a nuclear disaster.
- Managing waste and ensuring that it does not harm people or the environment.

Continue to the next page.

Nuclear Fusion

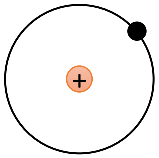
Nuclear fusion takes place when two light nuclei combine to form a larger, more stable nucleus.

Nuclear fusion can only occur if the two nuclei that are to be combined collide at high speed, overcoming the electrostatic repulsion and coming close enough to interact through the strong nuclear force.

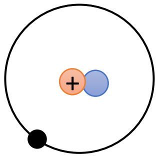


In the above diagram a deuterium nucleus fuses with a tritium nucleus to create a helium nucleus and a neutron, whilst releasing energy in the process.

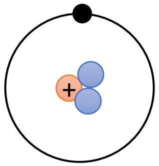
Deuterium and tritium are isotopes of hydrogen. Deuterium (^2_1H) is made up of one proton and one neutron. Tritium (^3_1H) is made up of one proton and two neutrons.



Hydrogen ^1_1H
1 Proton,
1 Electron,
0 Neutrons.



Deuterium ^2_1H
1 Proton,
1 Electron,
1 Neutrons.



Tritium ^3_1H
1 Proton,
1 Electron,
2 Neutrons.

Nuclear Fusion

The equation for this fusion reaction can be written as follows:

$$^2_1H + ^3_1H \rightarrow ^4_2He + ^1_0n + energy\ released$$

When you look at the graph of binding energy per nucleon against the nucleon number, you'll notice that the binding energy per nucleon of light nuclei (such as hydrogen, deuterium, and tritium) is significantly lower than that of bigger nuclei.

As a result, this difference in binding energy may be released if two light nuclei were forced to fuse into a single larger nucleus. This forms the basis of the process of nuclear fusion.

The mass of the larger nucleus formed by nuclear fusion is less than the sum of the masses of the fusing nuclei, and this mass difference is converted into energy.

Nuclear fusion between two hydrogen nuclei can only take place at extremely high temperatures. This gives them enough KE to fuse and overcome the electrostatic repulsion forces.

This is why fusion reactions take place at temperatures beyond 20 million degrees Celsius in the Sun and other stars.

Nuclear Fission/Fusion and Binding Energy

Summary

The differences in binding energy per nucleon propose a way to extract energy from nuclear reactions.

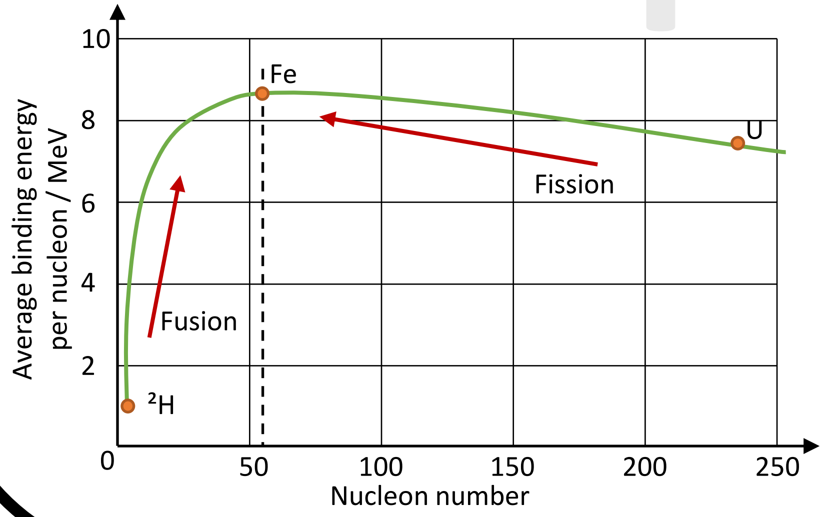
If a heavy nuclei can be split, lighter nuclei will form, and the difference in binding energy per nucleon is released as a result (nuclear fission).

Likewise, fusing two very light nuclei together will result in a more stable nucleus, and the difference in binding energy will be released (nuclear fusion).

In nuclear fission, energy is released because the new, smaller nuclei's have a higher average binding energy per nucleon.

Large nuclei are more likely to spontaneously fission because they are more unstable (the larger the nuclei the more unstable it is).

Nuclear fusion releases a lot of energy because the new heavier, nuclei's have a much higher average binding energy per nucleon.



Please see '**20.4.2 Nuclear Fission and Fusion worked examples'** pack for exam style questions.

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