A2 Level Physics

Chapter 8 – Further Mechanics

8.3.2 Simple Harmonic Motion

Worked Examples

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Simple Harmonic Motion Summary

Exam Style Question 1

(a) An object is oscillating with simple harmonic motion. Place a tick (✓) in the box against each true statement that applies to the acceleration of the object.

The acceleration ...
... is in the opposite direction to the displacement.
... is directly proportional to the amplitude squared.
... increases as the displacement decreases.
... increases as the speed of the object decreases

b) The graph in Fig. 3.1 shows the variation of the velocity v of the object with time t.

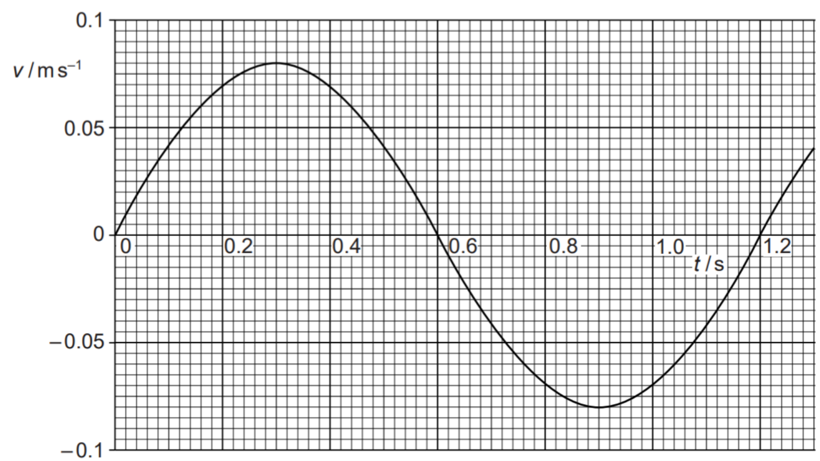

Fig. 3.1

Using the graph, determine
(i) the frequency of the motion
(ii) the amplitude of the motion
(iii) the maximum acceleration of the object.

Simple Harmonic Motion Summary

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The acceleration ...
✓ is in the opposite direction to the displacement.
... is directly proportional to the amplitude squared.
... increases as the displacement decreases.
✓ increases as the speed of the object decreases

b) The graph in Fig. 3.1 shows the variation of the velocity v of the object with time t.
Using the graph, determine
(i) the frequency of the motion

Use $f = \frac{1}{T}$

$$f = \frac{1}{1.2} = 0.83\ Hz$$

(ii) the amplitude of the motion

Use $v_{max} = (2\pi f)\ A$ and rearrange for A:

$$A = \frac{v_{max}}{2\pi f} = \frac{(0.08\ m\ s^{-1})}{2\pi(0.833 \ldots Hz)} = 0.015\ m$$

(iii) the maximum acceleration of the object.

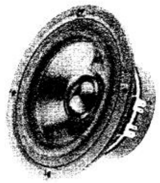
Use $a_{max} = (2\pi f)^2\ A$

$$a_{max} = [2\pi(0.833 \ldots Hz)]^2 (0.0152 \ldots\ m)$$
$$a_{max} = 0.42\ ms^{-2}$$

Simple Harmonic Motion Summary

Exam Style Question 2

The speaker shown below is used to produce the bass notes in a music system.



The cone moves with simple harmonic motion and it emits a single-frequency sound of 100 Hz. When it is producing a loud sound, the cone moves through a maximum distance of 2.0 mm.

The equation that mathematically describes the displacement of the cone is

$$x = 1.0 \times 10^{-3} \cos 628\, t.$$

a) Show that the data for this speaker lead to the numbers in the equation above.

b) Calculate

(i) the maximum acceleration of the cone

(ii) the maximum speed of the cone

c) On the grid below sketch the acceleration-time graph for two cycles of vibration of this speaker cone used under these conditions. Add suitable numerical scales to the two axes.

Simple Harmonic Motion Summary

Exam Style Question 2

a) Show that the data for this speaker lead to the numbers in the equation above.

Equation for shm is $x = A\cos\omega t$

We just need to find A and ω with the data that is given to us in the question.

We know that the maximum distance is 2.0 mm, so that is the distance from the crest to the trough so the amplitude is:

$$amplitude, A = \frac{2.0\ mm}{2} = 1.0mm$$

Therefore the amplitude (A) is $1.0mm$ or $1.0 \times 10^{-3}\ m$

We know $f = 100\ Hz$ so:

$$\omega = 2\pi f = 2\pi(100\ Hz)$$

$$\therefore\ \omega = 628.3185 \ldots = 628\ rad\ s^{-1}\ or\ 6.28 \times 10^{2}\ rad\ s^{-1}$$

b) Calculate

(i) the maximum acceleration of the cone

Use $a = \omega^2 A$

$$a = (628 rad\ s^{-1})^2\ (1 \times 10^{-3}\ m)$$

$$a = 394\ m\ s^{-2}$$

(ii) the maximum speed of the cone

Use $v = \omega A$

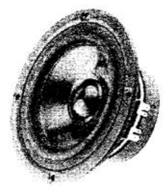
$$v = (628\ rad\ s^{-1})(1 \times 10^{-3}\ m)$$

$$v = 0.63\ m\ s^{-1}$$

Simple Harmonic Motion Summary

Exam Style Question 2

The speaker shown below is used to produce the bass notes in a music system.



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The equation that mathematically describes the displacement of the cone is

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Simple Harmonic Motion Summary

Exam Style Question 2

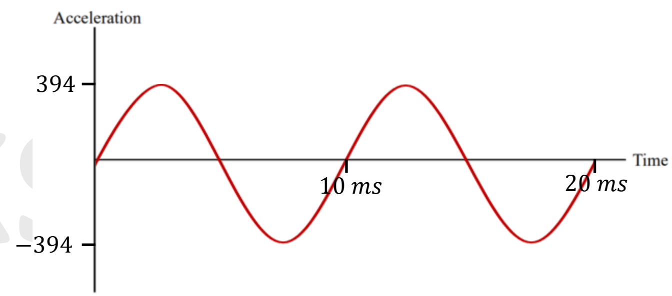
c) On the grid below sketch the acceleration-time graph for two cycles of vibration of this speaker cone used under these conditions. Add suitable numerical scales to the two axes.

First we need to find the time period so use:

$$T = \frac{1}{f} = \frac{1}{100\ Hz} = 0.01\ s = 10\ ms$$

We already know the amplitude (or the maximum acceleration) to be:

$$a_{max} = 394\ m\ s^{-2}$$



Simple Harmonic Motion Summary

Exam Style Question 3

(a) State two conditions concerning the acceleration of an oscillating object that must apply for simple harmonic motion.

(b) Fig. 3.1 shows how the potential energy, in mJ, of a simple harmonic oscillator varies with displacement.

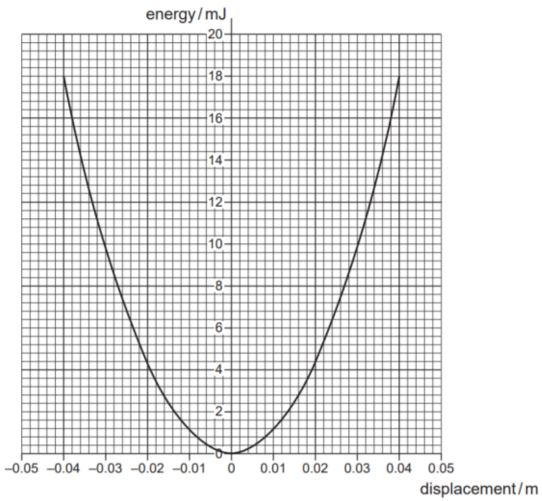


Fig. 3.1

On Fig. 3.1 sketch graphs to show the variation of

(i) kinetic energy of the oscillator with displacement – label this graph K,

(ii) the total energy of the oscillator with displacement – label this graph T.

(c) Use Fig. 3.1 to determine

(i) the amplitude of the oscillations

(ii) the maximum speed of the oscillator of mass $0.12\ kg$

(iii) the frequency of the oscillations.

Simple Harmonic Motion Summary

Exam Style Question 3

(a)State two conditions concerning the acceleration of an oscillating object that must apply for simple harmonic motion.

1) Acceleration is directly proportional to the displacement from the equilibrium position.
2) Acceleration is always directed towards the equilibrium.

(b)On Fig. 3.1 sketch graphs to show the variation of
(i) kinetic energy of the oscillator with displacement – label this graph K,
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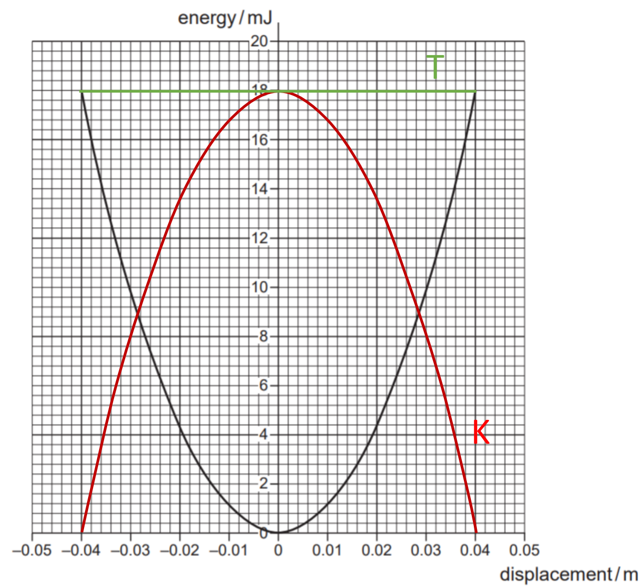

Fig. 3.1

Simple Harmonic Motion Summary

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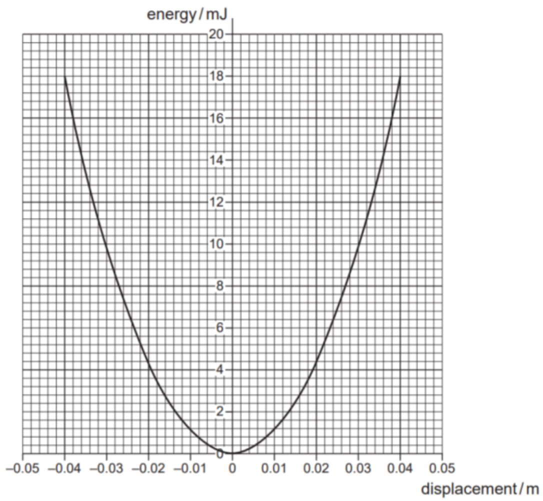

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(ii) the maximum speed of the oscillator of mass $0.12\ kg$

(iii) the frequency of the oscillations.

Simple Harmonic Motion Summary

Exam Style Question 3

(c) Use Fig. 3.1 to determine

(i) the amplitude of the oscillations

$$A = 0.04\ m$$

(ii) the maximum speed of the oscillator of mass $0.12\ kg$

Use $KE = \frac{1}{2}mv^2$ and rearrange for v

$$v = \sqrt{\frac{KE}{\frac{1}{2}m}} = \sqrt{\frac{18 \times 10^{-3}\ J}{\left(\frac{1}{2}\right)(0.12\ kg)}} = 0.54772 \ldots = 0.55\ m\ s^{-1}$$

(iii) the frequency of the oscillations.

Use $v_{max} = (2\pi f)\ A$

$$f = \frac{v_{max}}{2\pi A} = \frac{0.5477 \ldots ms^{-1}}{2\pi(0.04\ m)} = 2.1793 \ldots = 2.2\ Hz$$

Simple Harmonic Motion Summary

Exam Style Question 4

(a) Define simple harmonic motion.

(b) (i) A 120 g mass performs simple harmonic motion when suspended from a spring that has a spring constant of 3.9 $N\ m^{-1}$. Calculate the period T.

(ii) The simple harmonic motion is started by displacing the mass 15 cm from its equilibrium position and then releasing it. Calculate the maximum speed of the mass.

(iii) Calculate the maximum acceleration of the mass.

(iv) The 120 g mass is replaced by a wooden block. When the block performs simple harmonic motion the period is 1.4 s. Calculate the mass of the block.

Simple Harmonic Motion Summary

Exam Style Question 4

(a)Define simple harmonic motion.
An oscillation in which the acceleration of an object is:
1)Directly proportional to its displacement from its equilibrium position, and
2)Always in the opposite direction to the displacement.

(b) (i) Calculate the period T.

Use $T = 2\pi\sqrt{\frac{m}{k}}$

$$T = 2\pi\sqrt{\frac{(0.120\ kg)}{(3.9\ N\ m^{-1})}} = 1.10\ s$$

(ii) Calculate the maximum speed of the mass.
Use $v_{max} = (2\pi f)x_0$
But first calculate f:

$$f = \frac{1}{T} = \frac{1}{1.1021\ ...\ s} = 0.907\ ...\ Hz$$

Now sub f to find v_{max}

$$v_{max} = [2\pi(0.907\ ...\ Hz)](0.15\ m)$$
$$v_{max} = 0.86\ m\ s^{-1}$$

(iii) Calculate the maximum acceleration of the mass.
Use $a_{max} = (2\pi f)^2 x_0$

$$a_{max} = [2\pi(0.907\ ...\ Hz)]^2(0.15\ m)$$
$$a_{max} = 4.9\ m\ s^{-2}$$

(iv) Calculate the mass of the block.
We know that $T \propto \sqrt{m}$

$$T = k\sqrt{m}$$
$$\therefore\ k = \frac{T}{\sqrt{m}} = \frac{1.10\ s}{\sqrt{0.120\ kg}} = 3.175\ ...$$
$$T = (3.175\ ...)\sqrt{m}$$

Rearrange for m:

$$m = \left(\frac{T}{(3.175\ ...)}\right)^2 = \left(\frac{1.4\ s}{(3.175\ ...)}\right)^2 = 0.19\ kg$$

Simple Harmonic Motion Summary

Exam Style Question 5

Fig. 3.1 shows a metal plate attached to the end of a spiral spring. The end A of the spring is fixed to a rigid clamp. The plate is pulled down by a small amount and released. The plate performs simple harmonic motion in a vertical plane at a natural frequency of $8\ Hz$ and the spring remains in tension at all times.

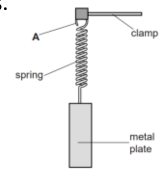


Fig. 3.1

(a) (i) On Fig. 3.2 sketch an acceleration a against displacement x graph for the motion of the metal plate. You are not required to give values on the axes.

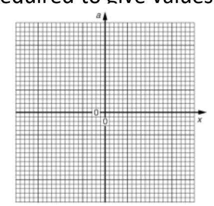


Fig. 3.2

(ii) Explain how your graph could be used to determine the frequency of oscillation of the metal plate.

(b) Fig. 3.3 shows the variation of the vertical velocity v of the plate with time t at a frequency of $8\ Hz$.

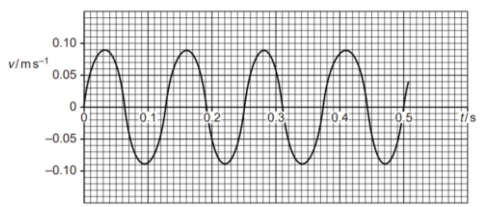


Fig. 3.3

Use the graph to determine

(i) the amplitude of the motion

(ii) the maximum vertical acceleration of the plate.

Simple Harmonic Motion Summary

Exam Style Question 5

(a)((i) On Fig. 3.2 sketch an acceleration a against displacement x graph for the motion of the metal plate. You are not required to give values on the axes.

We know that $F = -kx$ where $k = constant$ and $F = ma$

$$\therefore ma = -kx$$

$$a = -\frac{k}{m}x$$

$$a \propto -x$$

Therefore we would see a straight line graph going through the origin whilst having a negative gradient:

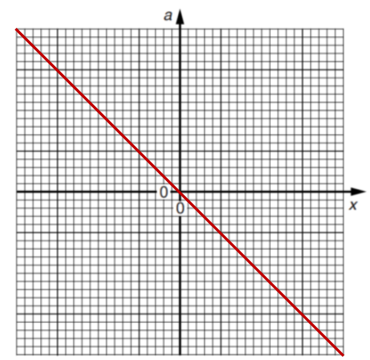


Fig. 3.2

(ii) Explain how your graph could be used to determine the frequency of oscillation of the metal plate.

Remember: $a = -(2\pi f)^2\, x$

$$-\frac{a}{x} = (2\pi f)^2$$

But from the graph we know: $-\frac{a}{x} = gradient$

$$\therefore gradient = (2\pi f)^2$$

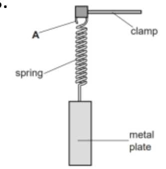
$$\sqrt{gradient} = 2\pi f$$

$$f = \frac{\sqrt{gradient}}{2\pi}$$

Simple Harmonic Motion Summary

Exam Style Question 5

Fig. 3.1 shows a metal plate attached to the end of a spiral spring. The end A of the spring is fixed to a rigid clamp. The plate is pulled down by a small amount and released. The plate performs simple harmonic motion in a vertical plane at a natural frequency of $8\ Hz$ and the spring remains in tension at all times.



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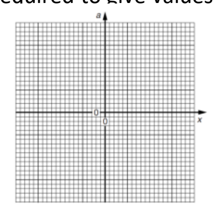

Fig. 3.2

(ii) Explain how your graph could be used to determine the frequency of oscillation of the metal plate.

(b) Fig. 3.3 shows the variation of the vertical velocity v of the plate with time t at a frequency of $8\ Hz$.

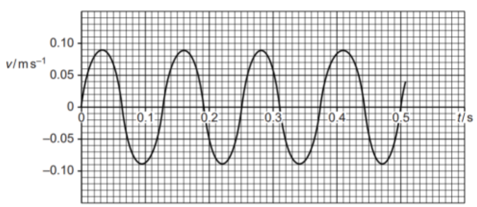

Fig. 3.3

Use the graph to determine

(i) the amplitude of the motion

(ii) the maximum vertical acceleration of the plate.

Simple Harmonic Motion Summary

Exam Style Question 5

(b) Use the graph to determine

(i) the amplitude of the motion

Use $v_{max} = (2\pi f)\ A$ and rearrange for A

$$A = \frac{v_{max}}{2\pi f} = \frac{0.09\ m\ s^{-1}}{2\pi(8\ Hz)} = 1.79049 \ldots \times 10^{-3}\ m$$

$$A = 1.8 \times 10^{-3}\ m$$

(ii) the maximum vertical acceleration of the plate.

Use $a_{max} = (2\pi f)^2\ A$

$$a_{max} = [2\pi(8\ Hz)\ ]^2 (1.790 \ldots \times 10^{-3}\ m)$$

$$a_{max} = 4.5\ m\ s^{-2}$$

Simple Harmonic Motion Summary

Exam Style Question 6

(a) A simple pendulum is given a small displacement from its equilibrium position and performs simple harmonic motion.

State what is meant by simple harmonic motion.

(b) (i) Calculate the frequency of the oscillations of a simple pendulum of length $984\ mm$.

(ii) Calculate the acceleration of the bob of the simple pendulum when the displacement from the equilibrium position is $42\ mm$.

Simple Harmonic Motion Summary

Exam Style Question 6

(a) State what is meant by simple harmonic motion.

An oscillation in which the acceleration of an object is:

1) Directly proportional to its displacement from its equilibrium position, and
2) Always in the opposite direction to the displacement.

(b) (i) Calculate the frequency of the oscillations of a simple pendulum of length $984\ mm$.

Use $T = 2\pi\sqrt{\frac{l}{g}}$

$$T = 2\pi\sqrt{\frac{984 \times 10^{-3}\ m}{9.81\ m\ s^{-2}}} = 1.9899\ldots s$$

Now use $f = \frac{1}{T}$ to find the frequency

$$f = \frac{1}{T} = \frac{1}{1.9899\ldots s} = 0.503\ Hz$$

(ii) Calculate the acceleration of the bob of the simple pendulum when the displacement from the equilibrium position is $42\ mm$.

Use $a = (2\pi f)^2\ x$

$$a = [2\pi(0.502\ldots Hz]^2(42 \times 10^{-3}\ m)$$

$$a = 0.42\ m\ s^{-2}$$

Please see '8.3.1 Simple Harmonic Motion notes' pack for revision notes.

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