



# AS Level Physics

## Chapter 3 – Forces and Motion

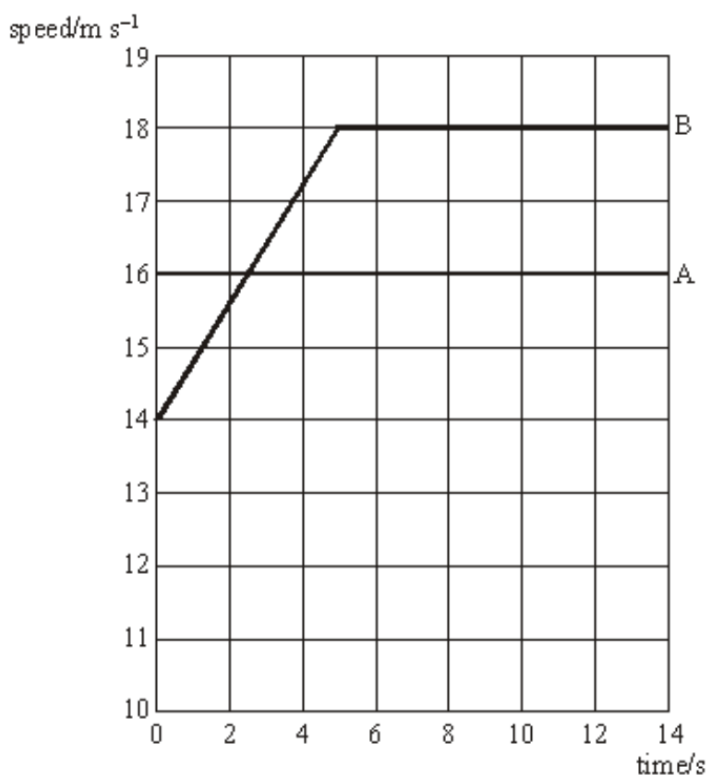
### 3.2.2 Linear Motion and Projectile Motion

#### Worked Examples

## Question

1

The graph represents the motion of two cars, A and B, as they move along a straight, horizontal road.



(a) Describe the motion of each car as shown on the graph.

(i) car A:

(ii) car B:

(b) Calculate the distance travelled by each car during the first 5.0 s.

(i) car A:

(ii) car B:

(c) At time  $t = 0$ , the two cars are level. Explain why car A is at its maximum distance ahead of B at  $t = 2.5$  s

# Question

1

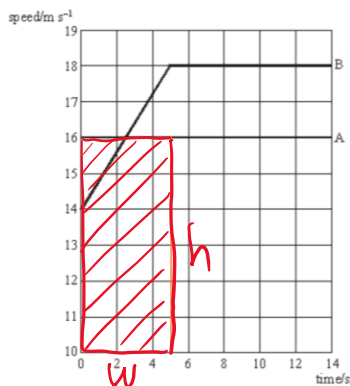
**a) Describe the motion of car A and Car B:**

- i) The graph for Car A shows that Car A is **travelling at constant speed**.
- ii) Car B accelerates for the first 5 seconds up to  $18 \text{ ms}^{-1}$  and then travels at a constant speed.

**b) Calculate the distance travelled by each car during the first 5.0s:**

**i) Car A:**

**Remember area under the speed time graph = distance**

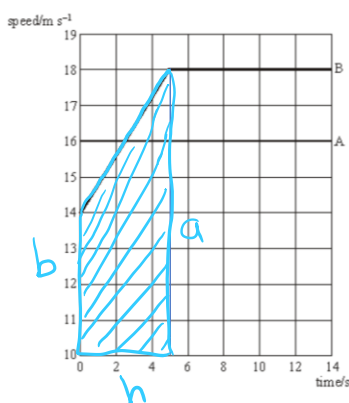


Therefore the distance travelled by Car A is represented by the red rectangular area.

$$\begin{aligned} \text{distance travelled by Car A} &= \text{area of the rectangle} \\ \text{distance} &= \text{width}(w) \times \text{height}(h) \\ \text{distance} &= 5.0 \text{ s} \times 16 \text{ ms}^{-1} \\ \text{distance} &= 80 \text{ m} \end{aligned}$$

**Therefore Car A travelled 80 m.**

**ii) Car B:**



The distance travelled by Car B is represented by the blue trapezium area.

$$\begin{aligned} \text{distance travelled by Car B} &= \text{area of the trapezium} \\ \text{distance} &= \frac{a + b}{2} \times h \\ \text{distance} &= \frac{18 \text{ ms}^{-1} + 14 \text{ ms}^{-1}}{2} \times 5 \text{ s} \\ \text{distance} &= 80 \text{ m} \end{aligned}$$

**Therefore Car B travelled 80m.**

**c) Explain why Car A is at its maximum distance ahead of B at  $t = 2.5$  seconds:**

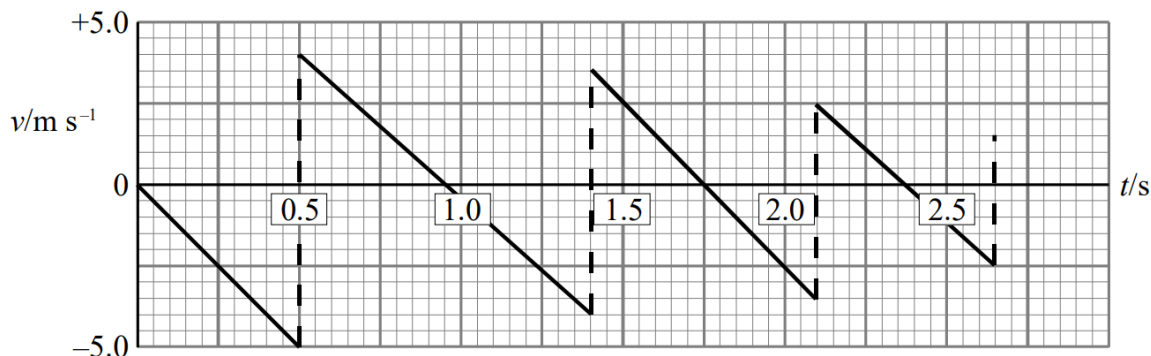
- Car B is initially at  $14 \text{ ms}^{-1}$  and Car A is at  $16 \text{ ms}^{-1}$ . So for the first 2.5 seconds car A is faster than car B.
- As Car A is faster than Car B for the first 2.5 seconds, the separation between car A and car B is increasing. Until  $t = 2.5 \text{ s}$  when car B reaches the same speed as car A and the separation between the cars is no longer increasing and is constant.
- This means car A has reached its maximum distance ahead of car B at  $t = 2.5 \text{ s}$ .
- After  $t = 2.5 \text{ s}$  car B is slowly getting faster than car A and so the separation between the two cars decreases because car B is catching up to car A.



## Question

2

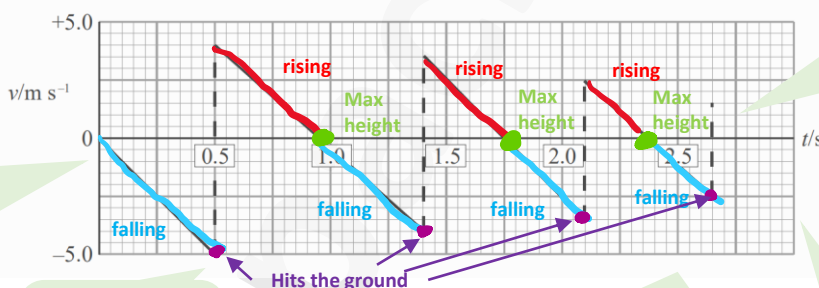
The diagram shows a velocity-time graph for a ball bouncing vertically on a hard surface. The ball was dropped at  $t = 0$  s.



- At what time does the graph show the ball in contact with the ground for the third time?
- The downward sloping lines on the graph are straight and parallel with each other. Why?

**a) At what time does the ball come in contact with the ground for the third time:**

3) The blue line shows that the ball is falling down. Red lines show that the ball is rising. The green dots show that the ball has reached its max height. The purple dots mean that the ball hits the ground.



4) The lines are decreasing in length each bounce because kinetic energy is lost with each bounce.

5) If all the lines were the same length then there is no energy loss.

1) To make this question easier to answer I have labelled the diagram showing the motion of the bouncing ball.

2) This graph shows that downwards (falling) is negative and upwards (rising) is positive.

Therefore at  $t = 2.1$  seconds is when the ball came into contact with the ground for the third time.

**b) Why are the downward sloping lines straight and parallel to each other:**

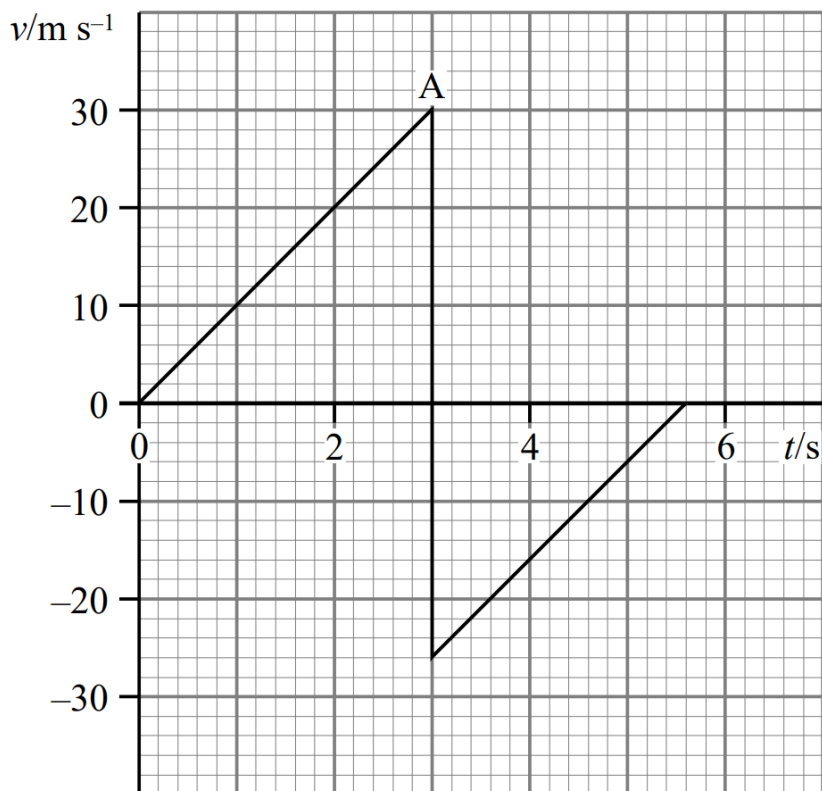
The lines represent the acceleration of the ball and show that the acceleration on the ball is constant.



## Question

3

A ball is dropped from a high window onto a concrete floor. The velocity–time graph for part of its motion is shown.



- Calculate the gradient from the origin to A.
- Comment on the significance of your answer.
- What happened to the ball at point A?
- Calculate the height of the window above the ground.



a) Calculate the gradient from the origin to point A:

$$\begin{aligned} \text{gradient} &= \frac{y_2 - y_1}{x_2 - x_1} \\ \text{gradient} &= \frac{30 - 0}{3 - 0} \\ \text{gradient} &= 10 \end{aligned}$$

Point A has coordinates  $(x_2, y_2) = (3, 30)$  and the origin has coordinates  $(x_1, y_1) = (0, 0)$

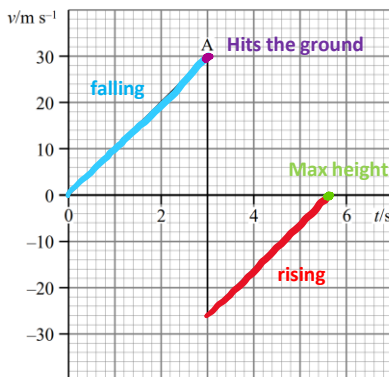
Therefore the gradient from the origin to point A is 10.

b) Comment on the significance of your answer:

The gradient of the graph represents the acceleration of the ball. The acceleration calculated is close to free fall or acceleration due to gravity.

Acceleration due to gravity or free fall is  $9.81 \text{ ms}^{-2}$ .

c) What happened to the ball at point A:



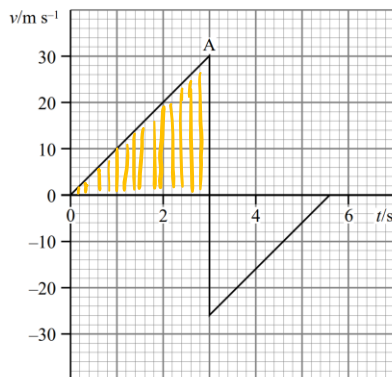
1) This graph shows that downwards (falling) is positive and upwards (rising) is negative.

2) The blue line shows that the ball is falling down. Red lines show that the ball is rising. The green dots show that the ball has reached its max height. The purple dots mean that the ball hits the ground.

3) As all the lines are the same length that means no energy is lost.

So at point A the ball hits the ground and the ball bounces.

d) Calculate the height of the window above the ground:



Area under the graph = displacement  
Area = area of triangle =  $\frac{1}{2} \times \text{base} \times \text{height}$

$$\text{Area} = \frac{1}{2} \times 3 \text{ s} \times 30 \text{ m}$$

$$\text{Area} = 45$$

Therefore the height of the window above the ground is 45 m.

Remember that the area under the velocity-time graph represents displacement. So to calculate the height of the window just calculate the orange area.

## Question

4

- a) Fig. 2.1 shows a graph of velocity against time for an object travelling in a straight line.

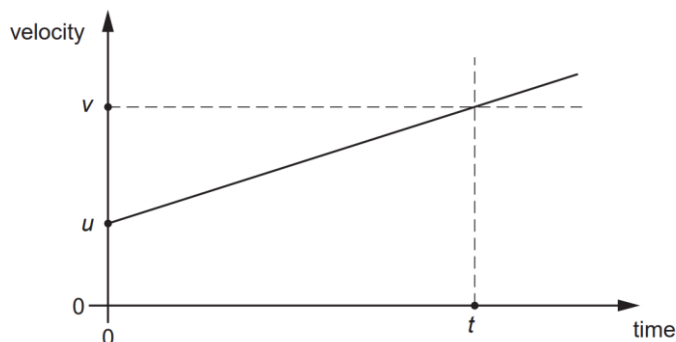


Fig. 2.1

The object has a constant acceleration  $a$ . In a time  $t$  its velocity increases from  $u$  to  $v$ .

- (a) Describe how the graph of Fig. 2.1 can be used to determine
- the acceleration  $a$  of the object. In your answer, you should use appropriate technical terms, spelled correctly.
  - the displacement  $s$  of the object.
- (b) Use the graph of Fig. 2.1 to show that the displacement  $s$  of the object is given by the equation:

$$s = ut + \frac{1}{2}at^2$$

- (c) In order to estimate the acceleration  $g$  of free fall, a student drops a large stone from a tall building. The height of the building is known to be 32 m. Using a stopwatch, the time taken for the stone to fall to the ground is 2.8 s.
- Use this information to determine the acceleration of free fall.
  - One possible reason why your answer to (c)(i) is smaller than the accepted value of  $9.81 \text{ ms}^{-2}$  is the reaction time of the student. State another reason why the answer is smaller than  $9.81 \text{ ms}^{-2}$ .



## Question

4

**ai) How can the graph be used to determine the acceleration (a):**

Acceleration (a) can be determined by calculating the gradient of the line.

$$\text{Acceleration}(a) = \text{gradient} = \frac{v - u}{t}$$

**aii) How can the graph be used to determine the displacement (s):**

The area under the graph is equal to the displacement (s).

**b) Prove:**

Split the graph into two different areas:

- A) Triangle
- B) Rectangle

And find the area and add them together.

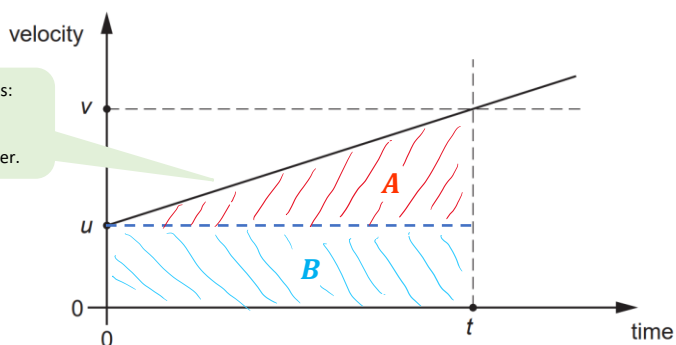


Fig. 2.1

Area of the rectangle(B) = height  $\times$  width =  $ut$

$$\text{Area of the triangle}(A) = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times t \times (v - u)$$

We know from equation 1  $v = u + at$  therefore  $v - u = at$  so:

$$\text{Area of a triangle} = \frac{1}{2} \times t \times at = \frac{1}{2}at^2$$

To get the total displacement we add the area of the triangle and the rectangle together to give:

$s = \text{area of the rectangle} + \text{area of the triangle}$

$$s = ut + \frac{1}{2}at^2$$

**ci) Use the information to determine the the acceleration of free fall:**

$$s = \text{displacement} = 32 \text{ m}$$

$u = \text{initial velocity} = 0 \text{ ms}^{-1}$  (because the stone is held at rest by the student before dropping it)

$v = \text{final velocity} = \text{we haven't been given this information}$

$a = \text{acceleration} = ? \text{ ms}^{-2}$  (what we have to find out)

$$t = \text{time} = 2.8 \text{ s}$$

$$s = ut + \frac{1}{2}at^2$$

$$32 \text{ m} = (0 \text{ ms}^{-1})(2.8 \text{ s}) + \frac{1}{2}(a)(2.8 \text{ s})^2$$

$$a = \frac{2 \times 32}{(2.8)^2}$$

$$a = 8.16 \text{ ms}^{-2}$$

**cii) State another reason why the answer is smaller than  $9.81 \text{ ms}^{-2}$ .**

The answer is smaller than  $9.81 \text{ ms}^{-2}$  because of air resistance/drag.

Always fill out the SUVAT with the information you have from the question. This makes picking the correct formula easier.

We use  $s = ut + \frac{1}{2}at^2$  because it doesn't include v.





## Question

5

- a) According to Aristotle (384 – 322 B.C.) ‘heavier objects fall faster than lighter ones’. Explain how one experiment carried out by Galileo (1564 – 1642) overturned Aristotle’s ideas of motion.
- b) Fig. 2.1 shows an arrangement used in the laboratory to determine the acceleration  $g$  of free fall.

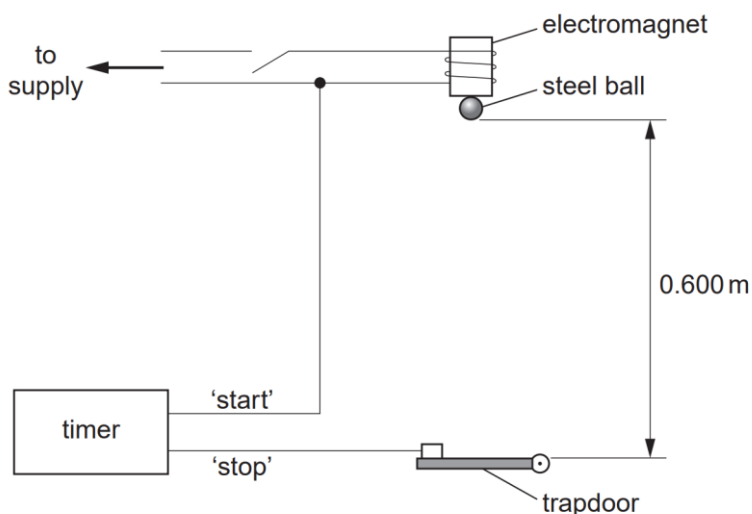


Fig. 2.1

The steel ball is held at rest by an electromagnet. When the electromagnet is switched off, the electronic timer is started and the ball falls. The timer is stopped when the ball opens the trapdoor. The distance between the bottom of the ball and the top of the trapdoor is 0.600 m. The timer records a time of fall of 0.356 s.

- Bi) Show that the value for the acceleration  $g$  of free fall obtained from this experiment is  $9.47 \text{ ms}^{-2}$ .
- Bii) State one reason why the experimental value in (i) is less than  $9.81 \text{ ms}^{-2}$ .
- Biii) On Fig. 2.2 sketch a graph to show the variation of the vertical distance  $s$  fallen by the ball with time  $t$ .

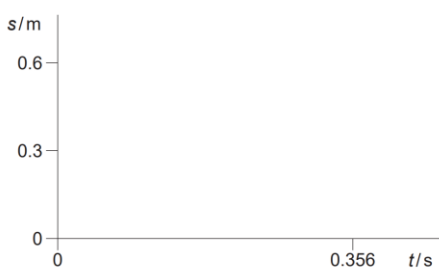


Fig. 2.2

**a) How did Galileo's experiment overturn Aristotle's idea of motion:**

Aristotle was an ancient Greek philosopher who was known for his famous theory that if two objects of different mass are dropped from the same height, the heavier object would always hit the ground before the lighter object.

Galileo was a scientist who overturned Aristotle's idea and thought all objects accelerate towards the ground at the same rate. This means that objects with different weights dropped from the same height should hit the ground at the same time. He reckoned that the reason why objects didn't do this was due to the existence of air resistance. To test his theory he did the incline plane experiment and his theory was correct.

So the answer is:

- Galileo rolled objects with different masses down an incline plane.
- He found the objects had the same acceleration.
- So, because they have the same acceleration the objects should hit the ground at the same time.

**bi) Show that the value of acceleration  $g$  is  $9.47 \text{ ms}^{-2}$ :**

$$s = \text{displacement} = 0.600 \text{ m}$$

$$u = \text{initial velocity} = 0 \text{ ms}^{-1} \text{ (because the ball is held at rest)}$$

$$v = \text{final velocity} = \text{we haven't been given this information}$$

$$a = \text{acceleration} = ? \text{ ms}^{-2} \text{ (what we have to find out)}$$

$$t = \text{time} = 0.356 \text{ s}$$

Always fill out the SUVAT with the information you have from the question. This makes picking the correct formula easier.

We use  $s = ut + \frac{1}{2}at^2$  because it doesn't include  $v$ .

$$s = ut + \frac{1}{2}at^2$$

$$0.600 \text{ m} = (0 \text{ ms}^{-1})(0.356 \text{ s}) + \frac{1}{2}(a)(0.356 \text{ s})^2$$

$$a = \frac{2 \times 0.600}{(0.356)^2}$$

$$a = 9.47 \text{ ms}^{-2}$$

**bii) State one reason why the experimental value in (i) is less than  $9.81 \text{ ms}^{-2}$ :**

The experimental value of  $9.47 \text{ ms}^{-2}$  is less than  $9.81 \text{ ms}^{-2}$  because of air resistance or drag. It could also be due to the delay between switching off the current and the ball being released (**sticky electromagnet**) and it **takes time to open the trapdoor**.

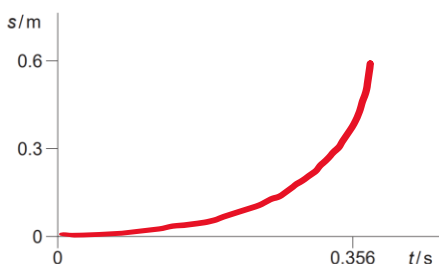
**biii) Sketch a graph to show the variation of the vertical distance  $s$  fallen by the ball with time  $t$ .**

Fig. 2.2

## Question

6

A small block of wood is held at a horizontal distance of 1.2 m from a metal ball. The metal ball is fired horizontally towards the block at a speed of  $8.0 \text{ ms}^{-1}$ . At the same instant the ball is fired, the block is released and it falls vertically under gravity.

Fig. 8.1 shows the paths of the metal ball and the block. The ball collides with the block. Air resistance is negligible.

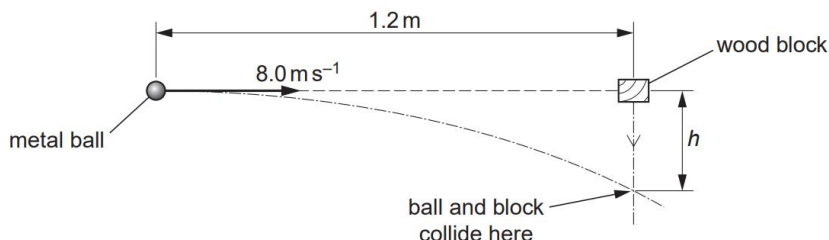


Fig. 8.1

- Show that the time between firing the ball and it colliding with the block is 0.15 s.
- Calculate the vertical distance  $h$  fallen by the wooden block when it collides with the metal ball.
- Briefly explain why the metal ball will always collide with the wood block, even if the speed of the ball or the horizontal distance is changed.

### a) Show that the time is 0.15 s.

This is the time taken for the metal block to travel a horizontal distance of 1.2m. Therefore there is no acceleration of free fall acting on the metal ball. So you can just use speed=distance/time.

$$\begin{aligned} \text{speed} &= \frac{\text{distance}}{\text{time}} \\ 8.0 \text{ ms}^{-1} &= \frac{1.2 \text{ m}}{t} \\ t &= \frac{1.2 \text{ m}}{8 \text{ ms}^{-1}} \\ \text{time} &= 0.15 \text{ s} \end{aligned}$$

### b) Calculate the vertical distance $h$ .

$$\begin{aligned} s &= ? \text{ m} \\ u &= 0 \text{ ms}^{-1} \\ v &= \text{not given} \\ a &= 9.81 \text{ ms}^{-2} \\ t &= 0.15 \text{ s} \end{aligned}$$

$$\text{Use } s = ut + \frac{1}{2}at^2$$

$$\begin{aligned} s &= (0)(0.15) + \frac{1}{2}(9.81)(0.15)^2 \\ s &= 0.11 \text{ m} \end{aligned}$$

Therefore the vertical distance  $h = 0.11 \text{ m}$

### c) Explain why the metal ball will always collide with the wood block, even if the speed of the ball or the horizontal distance is changed.

The ball will always collide with the wooden block even if the speed of the ball or the horizontal distance is changed because **they both have the same vertical acceleration of  $9.81 \text{ ms}^{-2}$ .**

## Question

7

Fig. 2.1 shows the path of water from a hose pipe.

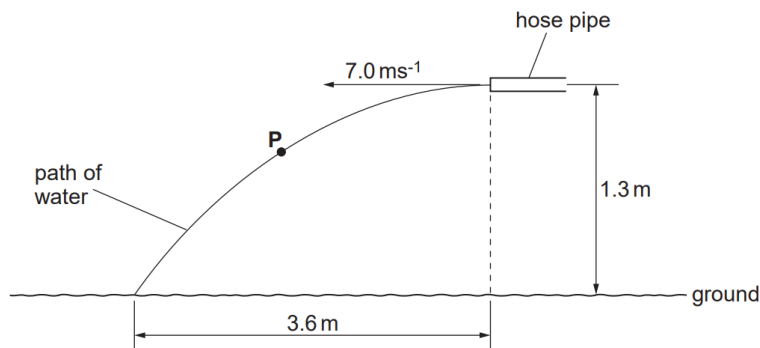


Fig. 2.1

The end of the horizontal hose pipe is at a height of 1.3 m from the ground. The initial horizontal velocity of the water is  $7.0 \text{ ms}^{-1}$ . The horizontal distance from the end of the hose pipe to the point where the water hits the ground is 3.6 m. You may assume that air resistance has negligible effect on the motion of the water jet.

- On Fig. 2.1, draw an arrow to show the direction of the acceleration of the water at point P. (Mark this arrow A).
- Explain why the horizontal component of the velocity remains constant at  $7.0 \text{ ms}^{-1}$ .
- Show that the water takes about 0.5 s to travel from the end of the pipe to the ground.
- Show that the speed of the water when it hits the ground is  $8.6 \text{ ms}^{-1}$ .

### a) Draw an arrow showing the direction of acceleration of the water:

Arrow A is the acceleration of free fall ( $9.81 \text{ ms}^{-2}$ ). This is the acceleration that causes the water particles to follow a projectile (curved) motion. The acceleration of free fall always act directly downwards.

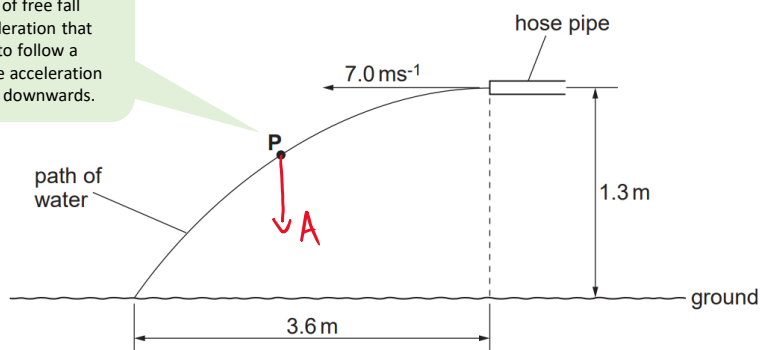


Fig. 2.1

### b) Why is the horizontal component not affected?:

**In the horizontal direction the acceleration of free fall is zero. Also the weight and the acceleration of free fall are at right angles to the motion and therefore have no effect on the horizontal component (or velocity).**



c) Show that the water takes 0.5 s to travel from the end of the pipe to the ground.

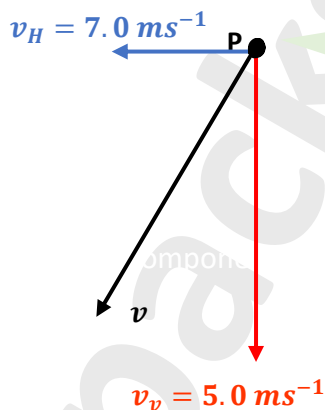
This is the time taken for the water to travel a horizontal distance of 3.6m. Therefore there is no acceleration of free fall acting on the water particles. So you can just use speed=distance/time.

$$\begin{aligned} \text{Use speed} &= \frac{\text{distance}}{\text{time}} \\ \text{time} &= \frac{\text{distance}}{\text{speed}} \\ \text{time} &= \frac{3.6 \text{ m}}{7.0 \text{ ms}^{-1}} \\ \text{time} &= \mathbf{0.514 \text{ s}} \end{aligned}$$

d) Show that the speed of the water when it hits the ground is  $8.6 \text{ ms}^{-1}$ :

$$\begin{aligned} s &= \text{don't need} \\ u &= 0 \text{ ms}^{-1} \\ v_v &= \text{vertical velocity} = ? \\ a &= 9.81 \text{ ms}^{-2} \\ t &= 0.5 \text{ s} \end{aligned}$$

$$\begin{aligned} \text{use } v &= u + at \\ v_v &= 0 + (9.81)(0.5) \\ \text{Therefore vertical velocity} &= 5.0 \text{ ms}^{-1} \end{aligned}$$



The information we have been given and worked out is just the **horizontal** and **vertical** velocities of the water particles (P). But the speed of the water when it hits the ground is the **resultant velocity (v)** of the horizontal and vertical components.

use Pythagoras theorem to find the resultant velocity:

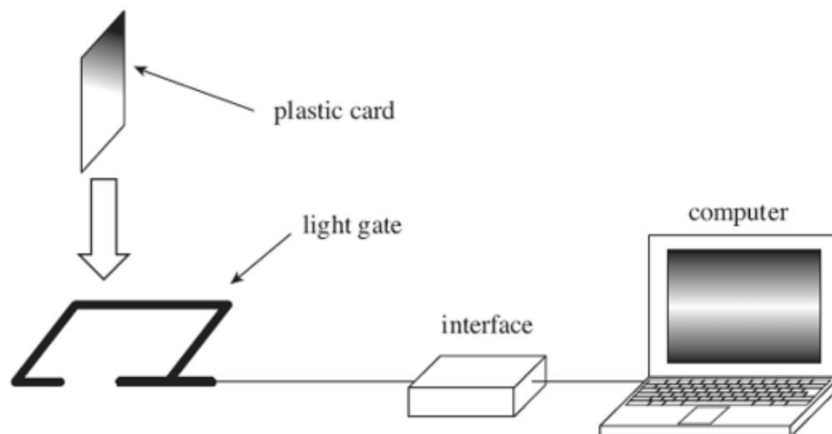
$$\begin{aligned} v^2 &= (7.0)^2 + (5.0)^2 \\ v &= \sqrt{(7.0)^2 + (5.0)^2} \\ v &= 8.6 \text{ ms}^{-1} \end{aligned}$$

Therefore the speed of the water before it hits the ground is  $8.6 \text{ ms}^{-1}$ .

## Question

8

A student measures the acceleration due to gravity,  $g$ , using the apparatus shown in the figure below. A plastic card of known length is released from rest at a height of 0.50m above a light gate. A computer calculates the velocity of the card at this point, using the time for the card to pass through the light gate.



- (a) The computer calculated a value of  $3.10 \text{ ms}^{-1}$  for the velocity of the card as it travelled through the light gate. Calculate a value for the acceleration due to gravity,  $g$ , from these data.
- (b) State and explain one reason why the card would give more reliable results than a table tennis ball for this experiment.

### a) Calculate the value of acceleration due to gravity (or free fall):

$$\begin{aligned} s &= 0.50 \text{ m} \\ u &= 0 \text{ ms}^{-1} \\ v &= 3.10 \text{ ms}^{-1} \\ a &=? \\ t &= \text{not given} \end{aligned}$$

$$\begin{aligned} \text{Use } v^2 &= u^2 + 2as \\ (3.10)^2 &= (0)^2 + 2a(0.50) \\ a &= \frac{(3.10)^2}{2 \times 0.50} \\ a &= 9.61 \text{ ms}^{-2} \end{aligned}$$

This experiment is similar to the trapdoor and electromagnet experiment shown in Question 1.

**So the acceleration due to gravity is  $9.61 \text{ ms}^{-2}$ .**

### b) Give one reason why using a card gives more reliable results compared to using a table tennis ball:

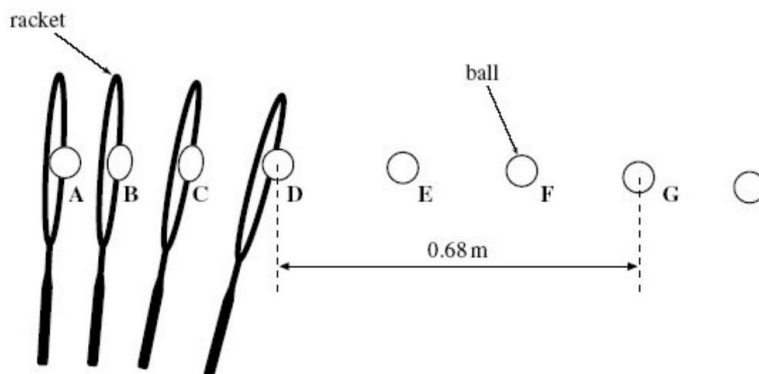
**The card will have almost constant acceleration whereas the table tennis ball won't because the air resistance affects the card less.**

The reason why air resistance affects the card less is because the card is more streamline (how easily an object can cut through the air) compared to the tennis ball so the card cuts right through the air whereas the ball will slow down due to air resistance.

# Question

9

A digital camera was used to obtain a sequence of images of a tennis ball being struck by a tennis racket. The camera was set to take an image every 5.0 ms. The successive positions of the racket and ball are shown in the diagram below.



- (a) The ball has a horizontal velocity of zero at A and reaches a constant horizontal velocity at D as it leaves the racket. The ball travels a horizontal distance of 0.68 m between D and G.
- (i) Show that the horizontal velocity of the ball between positions D and G in the diagram above is about  $45 \text{ ms}^{-1}$ .
- (ii) Calculate the horizontal acceleration of the ball between A and D.
- (b) At D, the ball was projected horizontally from a height of 2.3 m above level ground.
- (i) Show that the ball would fall to the ground in about 0.7 s.
- (ii) Calculate the horizontal distance that the ball will travel after it leaves the racket before hitting the ground. Assume that only gravity acts on the ball as it falls.
- (iii) Explain why, in practice, the ball will not travel this far before hitting the ground.

**ai) Show that the balls horizontal velocity is  $45 \text{ ms}^{-1}$  positions D and G:**

$$\text{Use velocity} = \frac{\text{displacement}}{\text{time}}$$

$$\text{time between D to G} = 15 \text{ ms} = 0.015 \text{ s}$$

$$\text{velocity} = \frac{0.68 \text{ m}}{0.015 \text{ s}}$$

$$\text{velocity} = 45 \text{ ms}^{-1}$$

**Therefore the horizontal velocity is  $45 \text{ ms}^{-1}$ .**

**aii) Calculate the horizontal acceleration of the ball between A and D:**

$$a = \frac{\Delta v}{\Delta t} = \frac{v - u}{\Delta t}$$

$$a = \frac{45.3 - 0}{0.015}$$

$$a = 3022 \text{ ms}^{-2}$$

**Therefore the horizontal acceleration is  $3022 \text{ ms}^{-2}$ .**

The time is 15 milliseconds (ms) because the camera takes images every 5 ms and there are three images between D and G

The final velocity is 45.3 m/s because at point D the ball leaves the racket and travels at a constant horizontal velocity which we calculated in part (ai). The initial velocity at point A is 0 m/s.

**Bi) Show that the ball would fall to the ground in about 0.7 s if it is projected at D:**

$$\begin{aligned}s &= 2.3 \text{ m} \\ u &= 0 \text{ ms}^{-1} \\ v &= \text{not needed} \\ a &= 9.81 \text{ ms}^{-2} \\ t &=?\end{aligned}$$

$$\begin{aligned}\text{Use } s &= ut + \frac{1}{2}at^2 \\ 2.3 &= (0)(t) + \frac{1}{2}(9.81)(t)^2 \\ 2.3 &= \frac{1}{2}(9.81)t^2 \\ \frac{2.3 \times 2}{9.81} &= t^2 \\ t &= \sqrt{\frac{2.3 \times 2}{9.81}} \\ t &= 0.68 \text{ seconds}\end{aligned}$$

**Therefore the time it takes the ball to fall to the ground is 0.68 seconds.**

**Bii) Calculate the horizontal distance the ball will travel after it leaves the racket:**

$$\begin{aligned}\text{speed} &= \frac{\text{distance}}{\text{time}} \\ \text{distance} &= \text{speed} \times \text{time} \\ \text{distance} &= 45.3 \times 0.685 \\ \text{distance} &= 30.6 \text{ m}\end{aligned}$$

There is no constant acceleration due to gravity acting on the horizontal component of the ball and so has no effect. Therefore you can just use the basic formula speed=distance/time.

**Therefore the horizontal distance the ball will travel is 30.6 m.**

**Biii) Why will the ball not travel this far before hitting the ground:**

When calculating the answer in (bii) we neglect the effects of air resistance. Therefore:

**Air resistance will cause horizontal deceleration and so the ball will not travel as far before hitting the ground.**

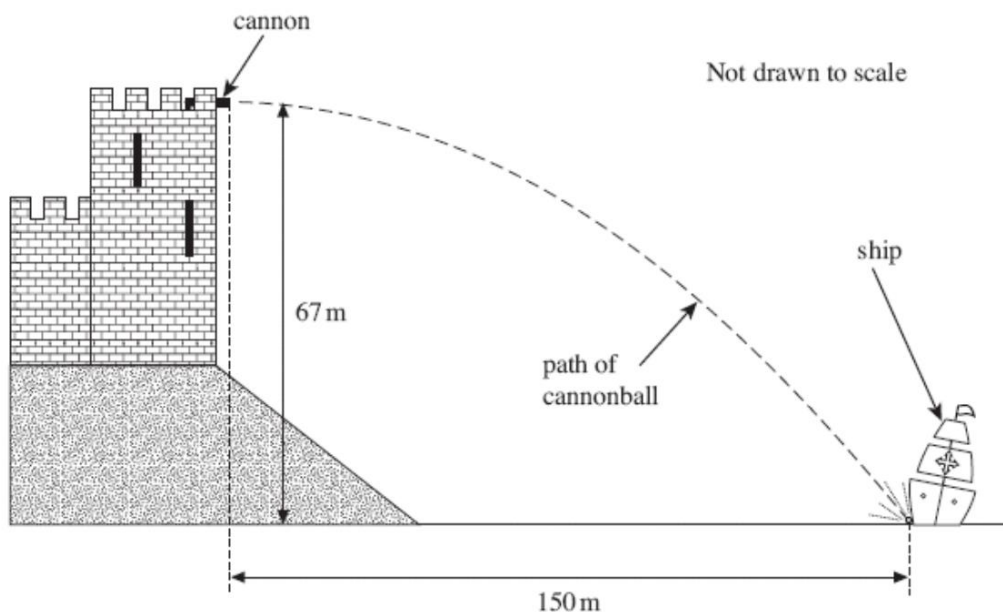




## Question

10

In a castle, overlooking a river, a cannon was once employed to fire at enemy ships. One ship was hit by a cannonball at a horizontal distance of 150 m from the cannon as shown in the figure below. The height of the cannon above the river was 67 m and the cannonball was fired horizontally.



- (a) (i) Show that the time taken for the cannonball to reach the water surface after being fired from the cannon was 3.7 s. Assume the air resistance was negligible.
- (ii) Calculate the velocity at which the cannonball was fired. Give your answer to an appropriate number of significant figures.
- (iii) Calculate the vertical component of velocity just before the cannonball hit the ship.
- (iv) By calculation or scale drawing, find the magnitude and direction of the velocity of the cannonball just before it hit the ship.



**ai) Show that the time taken for the cannonball to reach the water surface after being fired from the cannon was 3.7 s:**

$$\begin{aligned}s &= 67 \text{ m} \\ u &= 0 \text{ ms}^{-1} \\ v &= \text{not needed} \\ a &= 9.81 \text{ ms}^{-2} \\ t &=?\end{aligned}$$

$$\begin{aligned}s &= ut + \frac{1}{2}at^2 \\ 6.7 &= (0)t + \frac{1}{2}(9.81)(t^2) \\ t^2 &= \frac{6.7 \times 2}{9.81} \\ t &= \sqrt{\frac{6.7 \times 2}{9.81}} \\ t &= 3.696 \text{ s}\end{aligned}$$

Therefore the time taken for the cannonball to reach the water surface is 3.7s.

**a ii) Calculate the velocity at which the cannonball was fired:**

When the cannonball is fired, it is fired horizontally. There is no constant acceleration due to gravity acting on the horizontal component of the ball and so has no effect. Therefore you can just use the basic formula speed=distance/time.

$$\begin{aligned}v &= \frac{s}{t} = \frac{150 \text{ m}}{3.696 \text{ s}} = 40.586 \\ v &= 41 \text{ ms}^{-1}\end{aligned}$$

Therefore the cannonball was fired at 41 ms<sup>-1</sup>.

**Aiii) Calculate the vertical component of velocity just before the cannonball hits the ship:**

$$\begin{aligned}s &= \text{not needed} \\ u &= 0 \text{ ms}^{-1} \\ v &=? \\ a &= 9.81 \text{ ms}^{-2} \\ t &= 3.696 \text{ s}\end{aligned}$$

$$\text{Use } v = u + at$$

$$\begin{aligned}v &= (0) + (9.81)(3.696) = 36.257 \\ v &= 36 \text{ ms}^{-1}\end{aligned}$$

Therefore the vertical component just before the cannonball hits the ship is 36 ms<sup>-1</sup>.

**Aiv) Find the magnitude and direction of the velocity of the cannonball just before it hits the ship:**

use Pythagoras theorem to find the magnitude of resultant velocity:

$$\begin{aligned}v^2 &= (40.586)^2 + (36.257)^2 \\ v &= \sqrt{(40.586)^2 + (36.257)^2} \\ v &= 54 \text{ ms}^{-1}\end{aligned}$$

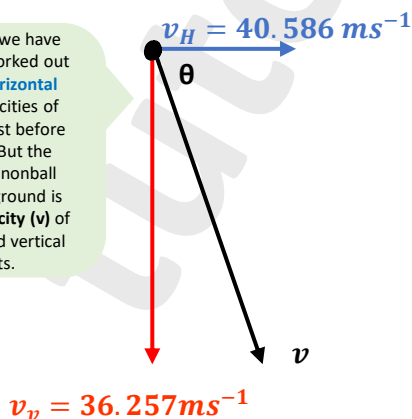
Therefore the magnitude of the cannonball before it hits the ground is 54 ms<sup>-1</sup>.

Use trigonometry to find the direction of the resultant velocity:

$$\begin{aligned}\tan \theta &= \frac{36.257}{40.586} \\ \theta &= \tan^{-1} \frac{36.257}{40.586} = 42^\circ\end{aligned}$$

Therefore the direction of the cannonball is 42° from the horizontal.

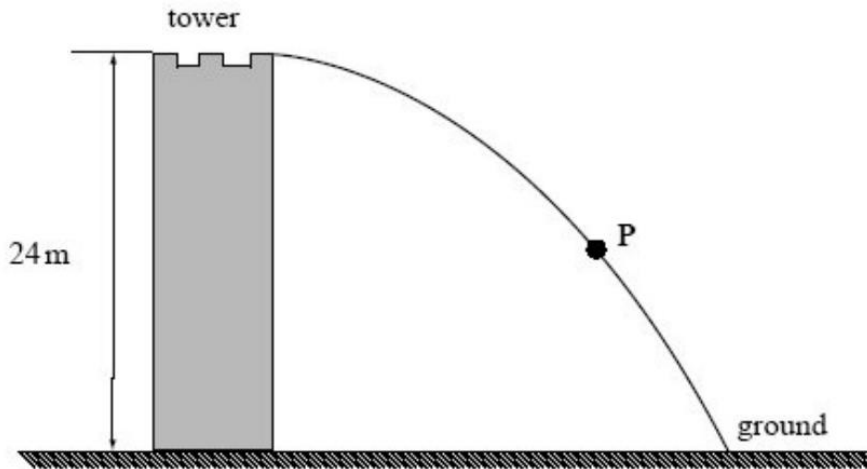
The information we have been given and worked out is just for the **horizontal** and **vertical** velocities of the cannonball just before it hits the ship. But the speed of the cannonball when it hits the ground is the **resultant velocity (v)** of the horizontal and vertical components.



# Question

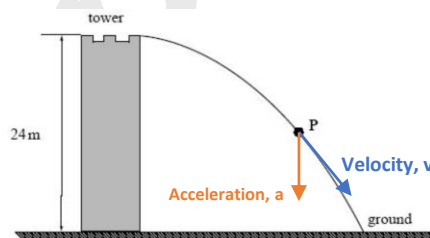
11

The diagram below shows the path of a ball thrown horizontally from the top of a tower of height 24 m which is surrounded by level ground.



- (a) Using two labelled arrows, show on the diagram above the direction of the velocity,  $v$ , and the acceleration,  $a$ , of the ball when it is at point P.
- (b) (i) Calculate the time taken from when the ball is thrown to when it first hits the ground. Assume air resistance is negligible.
- (ii) The ball hits the ground 27 m from the base of the tower. Calculate the speed at which the ball is thrown.

### ai) Show on the diagram the velocity and acceleration:



The velocity vector always acts tangential to the path and the acceleration is due to gravity (free fall) and so acts vertically downwards. Both from the centre of mass of the object.

### bi) Calculate the time taken for the ball to hit the ground:

$$s = 24 \text{ m}$$

$$u = 0 \text{ ms}^{-1}$$

$$v = \text{not needed}$$

$$a = 9.81 \text{ ms}^{-2}$$

$$t = ?$$

$$\text{Use } s = ut + \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2s}{g}} = \sqrt{\frac{2 \times 24}{9.81}}$$

$$t = 2.21 \text{ s}$$

### Bii) Calculate the speed at which the ball is thrown:

$$v = \frac{s}{t}$$

$$v = \frac{27}{2.21}$$

$$v = 12.2 \text{ ms}^{-1}$$

There is no constant acceleration due to gravity acting on the horizontal component of the ball and so has no effect. Therefore you can just use the basic formula speed=distance/time.



Please see the **'3.2.1 Linear Motion and Projectile Motion notes'** for revision notes.

For revision notes, tutorials, worked examples and more help visit [www.tutorpacks.co.uk](http://www.tutorpacks.co.uk).

