A2 Level Physics

Chapter 14 – Oscillations

14.1.1 Simple Harmonic Motion

Notes

Oscillations

Key Definitions

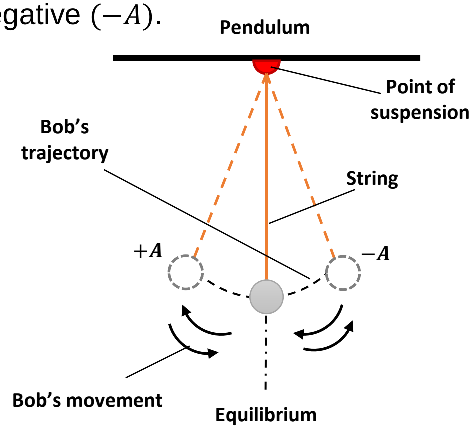
When an object moves back and forth frequently on either side of a fixed position (called the Equilibrium position), it is said to oscillate or vibrate.

The pendulum is an example of an oscillating system.

The motion a pendulum and many other oscillating systems can be represented by a displacement/time graph as shown in the diagram opposite which shows a sinusoidal (sin) graph.

One complete oscillation occurs when the pendulum moves from its equilibrium position (the midpoint of the pendulum's motion when displacement equals 0 or when the graph crosses the x-axis) to its maximum displacement (positive amplitude, $+A$) in one direction, back through the equilibrium position to the maximum displacement in the opposite direction (negative amplitude, $-A$), and back again to the equilibrium position. The displacement of an object is its distance from the equilibrium.

The positive and negative amplitude indicates the direction of the pendulum swing. In this scenario, when the pendulum swings to the left, the amplitude is positive $(+A)$, and when the pendulum swings to the right, the amplitude is negative $(-A)$.



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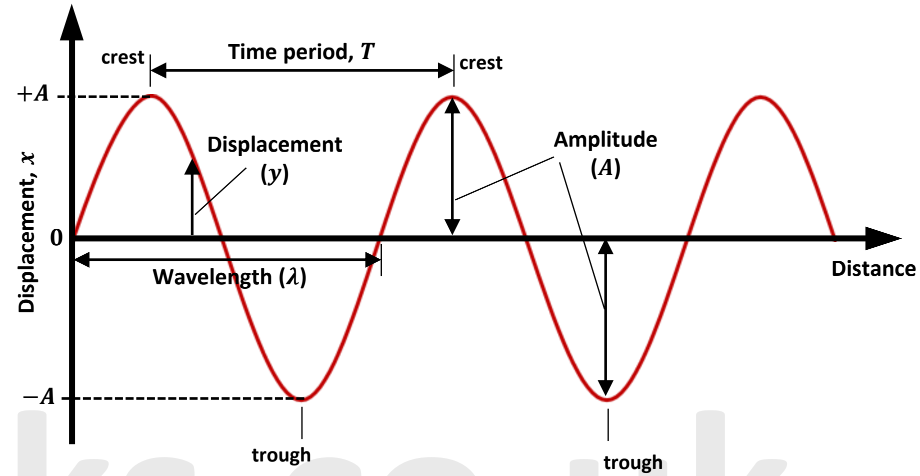
Oscillations

Key Definitions



- **Displacement (x) / metre (m)**

The distance and direction moved by any oscillating object in either direction from its equilibrium position (i.e. its rest position found on the horizontal axis where displacement is zero) at any given time.

- **Amplitude (A) / metre (m)**

The maximum displacement from its equilibrium position of any oscillating object. The amplitude is the height measured from the horizontal axis to the crest or trough.

- **Period (T) / seconds (s)**

The time taken to complete one full oscillation of an oscillating object.

- **Frequency (f) / Hertz (Hz)**

The number of complete waves passing a fixed point per second.

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Oscillations

Key Definitions

- **Phase Difference (ϕ) / radians, degrees or fractions of a cycle**

Phase difference is the amount by which one wave lags behind another wave.

- **Angular Frequency (ω) / $rad\ s^{-1}$**

Angular frequency, also known as circular frequency, is the angular displacement per unit time expressed in radians per second.

$$Angular\ frequency, \omega = 2\pi f$$

Note:

$$Frequency = \frac{1}{time\ period}$$

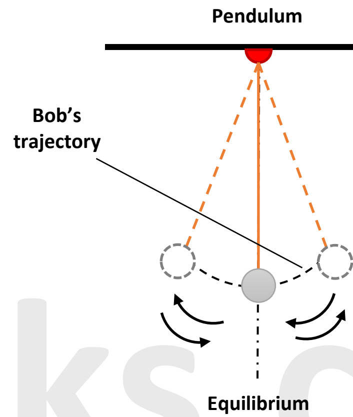
$$f = \frac{1}{T}$$

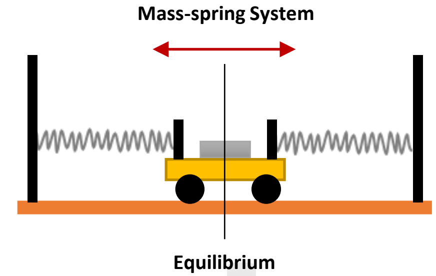
So we can write angular frequency as:

$$\omega = \frac{2\pi}{T}$$

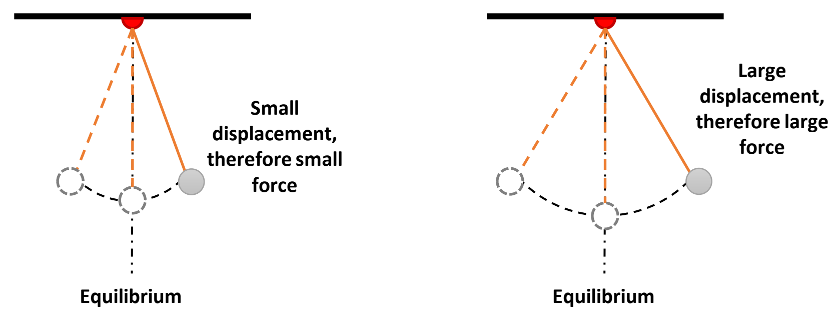
Simple Harmonic Motion

A pendulum or a mass-spring system are good examples of simple harmonic motion (SHM). SHM is when an object oscillates to and from either side of an equilibrium position. This equilibrium position is the midpoint of the objects motion. The displacement of an object is its distance from equilibrium.





There is always a restoring force present pulling or pushing the object back towards the equilibrium position. The size of the restoring force is proportional to the displacement. Its the restoring force that makes the object accelerate towards the equilibrium.



Simple Harmonic Motion

Simple Harmonic Motion (SHM) is defined as:

An oscillation in which the acceleration of an object is:

1) Directly proportional to its displacement from the equilibrium position,

2) Always in the opposite direction to the displacement.

This definition can be written as:

$$a \propto -x$$

Where a is the acceleration and x is the displacement. The minus sign indicates that the acceleration is in the opposite direction to the displacement.

SHM requires the following conditions:

- A particle/object vibrates about a fixed point.
- The acceleration is always directed towards that fixed point.
- The magnitude of the acceleration is proportional to the displacement from that fixed point.

SHM can be seen in the following examples:

- Simple pendulum.
- Mass spring system.
- A swing.

Simple Harmonic Motion

The general mathematical equation which defines SHM is:

$$a = -\omega^2 x = -(2\pi f)^2 x$$

Where:

a = acceleration in ms^{-2}

ω = angular frequency in $rads^{-1}$

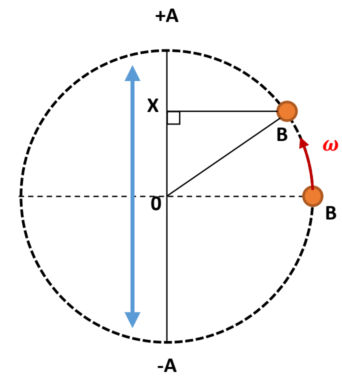
x = displacement in m

f = frequency in $hertz, Hz$.

The minus sign just indicates that the acceleration occurs in the opposite direction to the displacement.

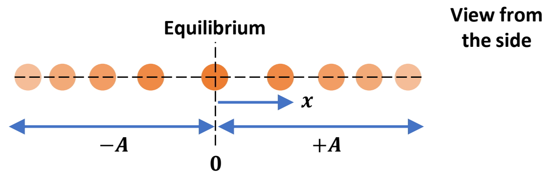
Before we go into the math behind SHM, it's important to understand that oscillations and circular motion are closely linked.

Consider a ball, B, moving in a circle with an angular frequency (ω). The ball moves from its starting position and completes 1 full revolution. The amplitude of the ball's SHM is the same as the radius of the circle. If you take the motion of the ball and plot the imaginary perpendicular line X onto the diameter $+A$ and $-A$, you can see that X performs a SHM which takes it from $0 \rightarrow +A \rightarrow 0 \rightarrow -A \rightarrow 0$.

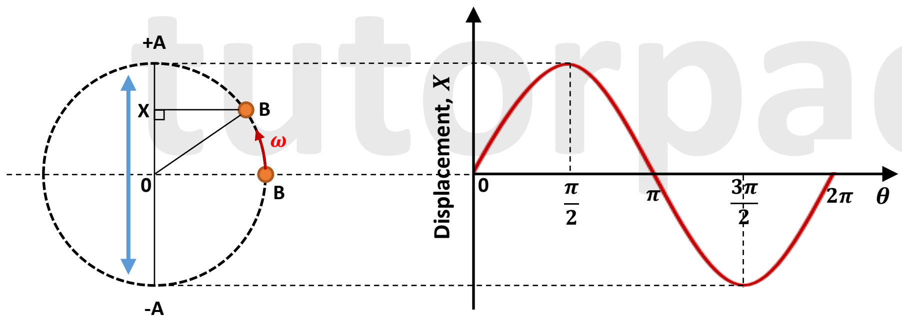


Simple Harmonic Motion

When you look at the same ball moving in a circle from a side view instead of the plan view (from above), it appears to be oscillating from side to side and moving with SHM.



Returning to our circular motion of the ball, we can obtain a sine curve by plotting the linear displacement (X) of the imaginary perpendicular line from B, against its angular displacement (θ).



$$angular\ frequency(\omega) = \frac{angular\ displacement\ (\theta)}{time\ taken\ (t)}$$

So, for 1 complete revolution of B, which is 1 complete oscillation of the imaginary perpendicular line X across $+A$ to $-A$.

Remember that 1 complete oscillation is equal to 2π therefore:

$$\omega = \frac{2\pi}{T}, \left(since\ T = \frac{1}{f}\right)$$
$$\omega = 2\pi f$$

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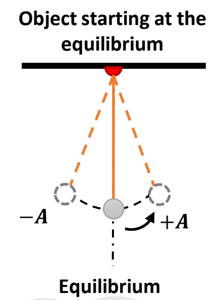
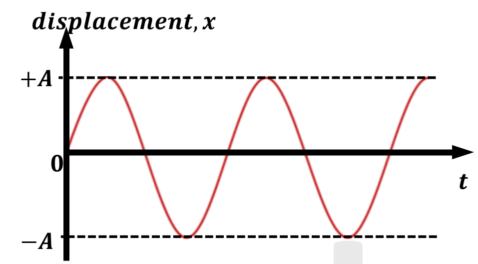
Simple Harmonic Motion

We know a sine curve can be obtained for an object moving in SHM. Now let's find out how we can draw graphs of displacement, velocity and acceleration of an object oscillating with SHM over time.

Displacement (x):

Displacement (x) varies as a sine or cosine wave with a maximum value, A (the amplitude).





If $x = 0$ when $t = 0$ means that the oscillation starts at the centre (i.e., equilibrium position) of the motion and the moves to the maximum displacement equal to $+A$. Therefore, the displacement (x) at time (t) is given by the equation for a sinusoidal oscillation:

$$x = Asin\theta$$

But we already know: $\omega = \frac{\theta}{t}, \therefore\ \theta = \omega t$. Substituting this to the above equation we get:

$$x = Asin(\omega t)$$

And since ω is also equal to $\omega = 2\pi f$, we can rewrite the equation as:

$$x = Asin(2\pi f t)$$

Where:

- x = displacement from the equilibrium position at any time (t) in m
- A = maximum displacement from the equilibrium position, amplitude in m
- ω = angular frequency in $rad\ s^{-1}$.
- f = frequency in Hz
- t = time in s

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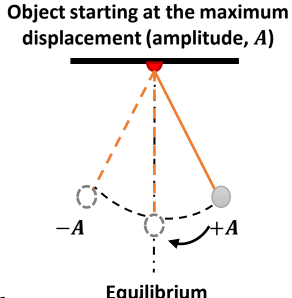
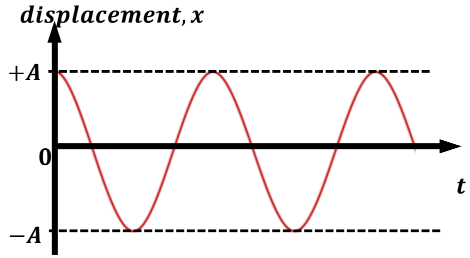
Simple Harmonic Motion

Displacement (x):





If $x = +A$ when $t = 0$ means the oscillation starts at the positive maximum displacement (i.e., positive amplitude, $+A$) of the motion and the object is approaching the equilibrium, then its displacement at time (t) can be calculated using the equation for a cosine oscillation:

$$x = A\cos\theta$$

But we already know: $\omega = \frac{\theta}{t}, \therefore\ \theta = \omega t$. Substituting this to the above equation we get:

$$x = A\cos(\omega t)$$

But because: $\omega = 2\pi f$ also, we can rewrite the equation as:

$$x = A\cos(2\pi f t)$$

Where:

- x = displacement from the equilibrium position at any time (t) in m
- A = maximum displacement from the equilibrium position, amplitude in m
- ω = angular frequency in $rad\ s^{-1}$.
- f = frequency in Hz
- t = time in s

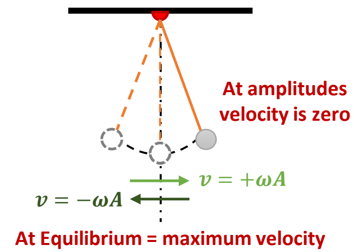
Note: The quantity $(2\pi f t)$ is in Radians, and so any calculations should be carried out with your calculator set to RAD.

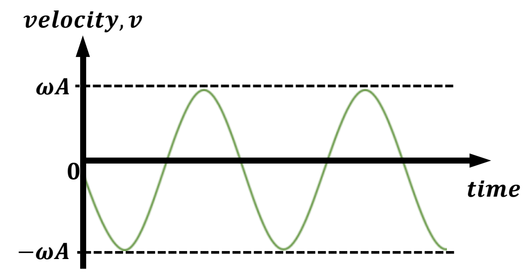
Simple Harmonic Motion

Velocity (v):

Velocity, v, is the gradient of the displacement – time graph $\left(i.e.\ v = \frac{dx}{dt}\right)$. It has a maximum value of ωA (where ω is the angular frequency of the oscillation).

Below is a graph of a SHM velocity-time graph of a pendulum starting at a positive amplitude:





The velocity of an object moving with SHM can be calculated using the following equation:

$$v = \pm\omega\sqrt{A^2 - x^2}$$

Where:

v = velocity in ms^{-1}

ω = angular frequency in $rad\ s^{-1}$

A = amplitude in m

x = displacement in m

The velocity is positive if the object is moving in the positive direction (e.g., to the right), and negative if it is moving in the negative direction (e.g., to the left) – hence the $\pm$ sign in the equation.

When the object is passing through the equilibrium, where $x = 0$, the object reaches maximum speed, and we get the equation:

$$v_{max} = \pm\omega A = \pm(2\pi f)A$$

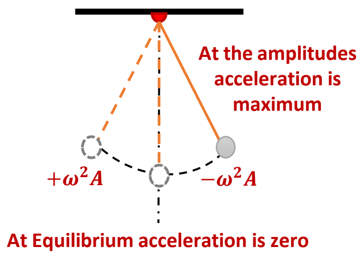
The velocity is 0 at maximum displacements (i.e., amplitudes).

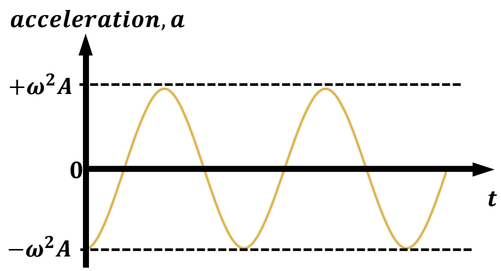
Simple Harmonic Motion

Acceleration (a):

Acceleration, a, is the gradient of the velocity-time graph $\left(i.e. \, a = \frac{dv}{dt}\right)$. It has a maximum value of $\omega^2 A$.

Below is a graph of a SHM acceleration-time graph of a pendulum starting at a positive amplitude:





The acceleration of an object moving with SHM can be calculated using the following equation:

$$a = -\omega^2 x$$

Where:

a = acceleration in ms^{-2}

ω = angular frequency in $rad\ s^{-1}$

x = displacement in m

Remember this equation from the previous pages, where we defined SHM.

The acceleration is positive if the object is moving in the negative direction (e.g., to the left), and negative if it's moving in the positive direction (e.g., to the right).

When the object is at its maximum displacement, where $x = \pm A$, the maximum acceleration occurs:

$$a = \pm\omega^2 A = \pm(2\pi f)^2 A$$

At the equilibrium (where $x = 0$) the acceleration is zero.

Simple Harmonic Motion Summary

Consider a pendulum undergoing SHM about an equilibrium position (0). The oscillation has a period (T) and an amplitude (A).

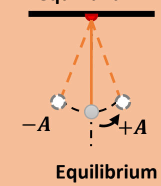
The below summarises the value of:

Displacement (x), velocity (v) and acceleration (a) at time (t) between 0 and T.

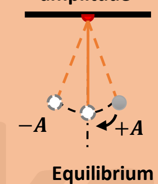
1) The pendulum is at its equilibrium position going towards the positive amplitude $(+A)$:

$$t = 0$$
$$x = 0$$
$$v = v_{max}(+ve)$$
$$a = 0$$

1) Object starting at the equilibrium



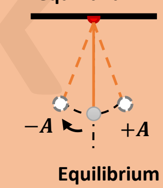
2) Object at the positive amplitude



2) The pendulum is at the positive amplitude $(+A)$:

$$t = T/4$$
$$x = +A$$
$$v = 0$$
$$a = -\omega^2 A$$

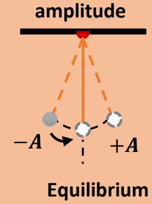
3) Object at the equilibrium



3) The bob returns back to the equilibrium position making its way towards the negative amplitude $(-A)$:

$$t = T/2$$
$$x = 0$$
$$v = v_{max}(-ve)$$
$$a = 0$$

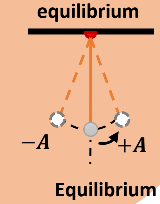
4) Object at the negative amplitude



4) This is the point where the bob is at the negative amplitude $(-A)$:

$$t = 3T/4$$
$$x = -A$$
$$v = 0$$
$$a = +\omega^2 A$$

5) Object at the equilibrium

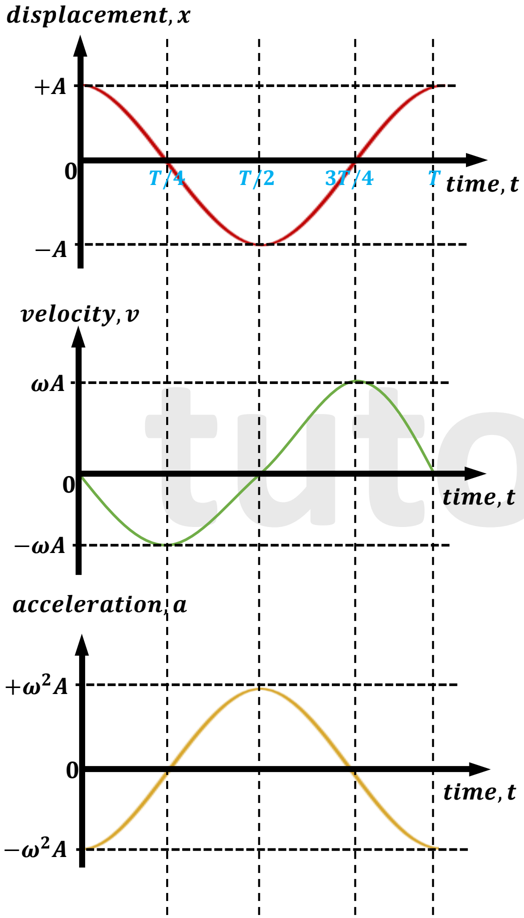


5) At this point the bob returns to the equilibrium position heading towards the positive amplitude $(+A)$. Here the pendulum has completed one full oscillation.:

$$t = T$$
$$x = 0$$
$$v = v_{max}(+ve)$$
$$a = 0$$

Simple Harmonic Motion Summary

Summary of displacement, velocity and acceleration graphs for an object undergoing SHM:



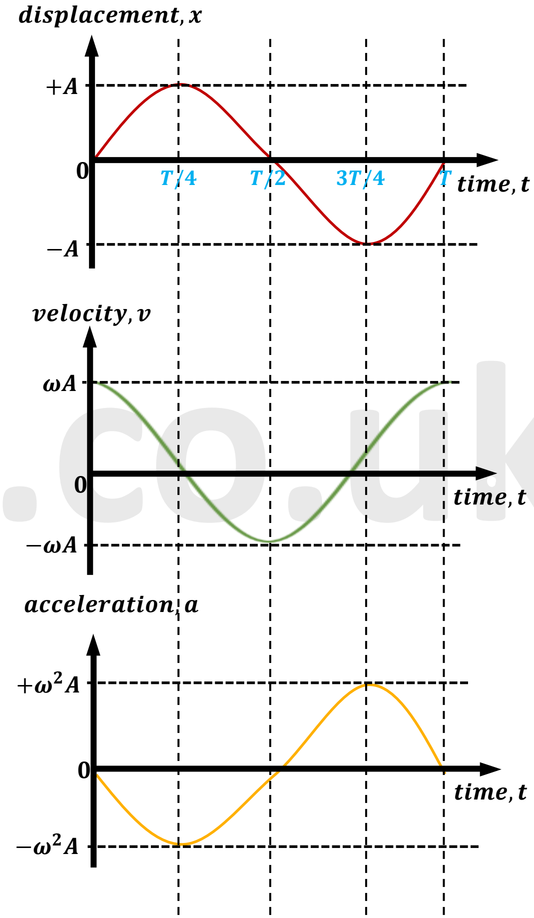
When you compare the x/t and a/t graph, you'll notice that they are both sine curves, but the a/t graph is inverted compared to the x/t graph. This means acceleration is always in the opposite direction to the displacement.

Therefore:
When $x = +A$, $a = -(2\pi f)^2 x$
When $x = -A$, $a = +(2\pi f)^2 x$
When $x = 0$, $a = 0$

Note: The Time period is independent of the amplitude of the oscillations.

Simple Harmonic Motion Summary

Summary of displacement, velocity and acceleration graphs for an object undergoing SHM when the oscillation starts at the equilibrium.



Observing Oscillations

Mass – spring system

A heavily-loaded trolley is attached by identical springs to two fixed stands.

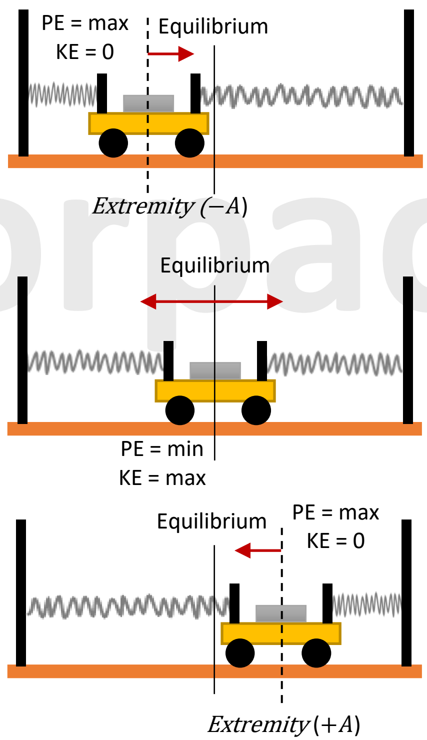
When the trolley is pulled horizontally to one side and then released, it appears to freely oscillate back and forth along the bench. Like a pendulum, this mass-spring system oscillates with simple harmonic motion, however instead of the object swinging, the trolley simply goes back and forth horizontally.

The springs attached to the trolley alternately stretch and compress as the trolley oscillates and we can see that the trolley's speed is:

- Greatest at the centre of the oscillation.
- Zero at the extremities (maximum displacement, amplitude) of the oscillation.

This means that at the centre of the oscillation, the kinetic energy of the system is a maximum and potential energy is a minimum.

At the extremities of the oscillation, the potential energy is a maximum and the kinetic energy is zero.



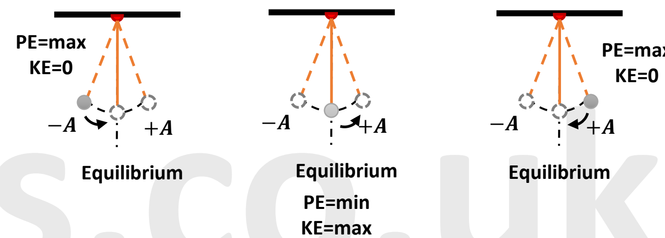
Observing Oscillations

Simple Pendulum

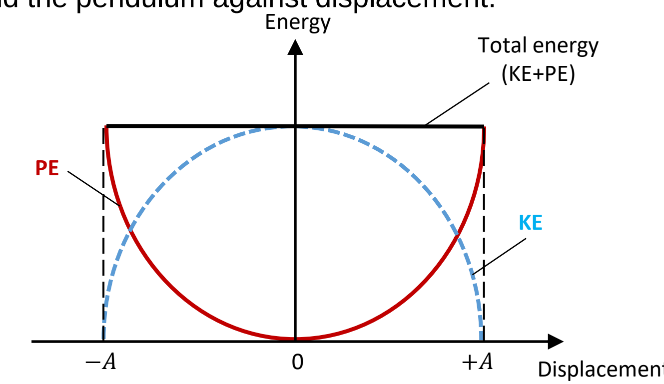
A pendulum can be set into oscillation when the pendulum bob is pulled slightly to one side and released.

The bob's speed is maximum at the centre of the oscillation and zero at the extremities.

As the pendulum oscillates about its equilibrium position, the kinetic energy of the system is maximum at the centre and zero at the extremities, while the potential energy is minimum at the centre and maximum at the extremities.



With this information, we can plot the energy of the mass-spring system and the pendulum against displacement:



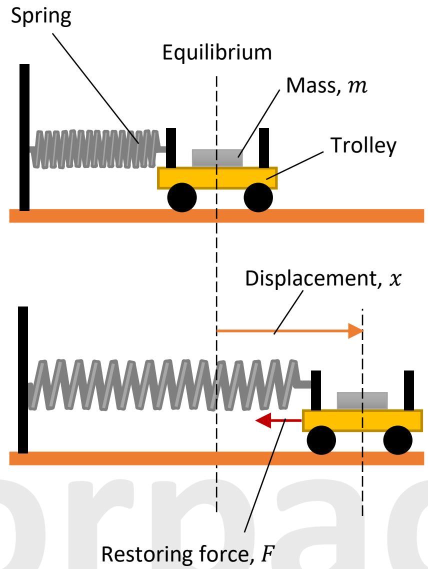
The total energy of the system at any point in the oscillation is the sum of the KE and PE at that point.

Investigating the mass-spring system

A mass spring system oscillates in SHM. When the mass is pushed or pulled either side of the equilibrium position, there's a restoring force exerted on it.

The size and direction of this restoring force is given by Hooke's law:

$$F = k\Delta L$$



For a displacement x, the restoring force becomes $F = -kx$. The sign is negative because the force acts in the opposite direction to the displacement – back towards the equilibrium position.

Using the above equation, we can also conclude that $F \propto x$. As a result, greater the displacement the greater the restoring force.

Furthermore, because the mass spring system oscillates in SHM, we may use the equation below to calculate the time period of oscillation:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Where:

T = time period of oscillation in s

m = mass in kg

k = spring constant in Nm^{-1}

Remember that the time period is the amount of time it takes for one complete oscillation (max left position, to max right position, back to max left position).

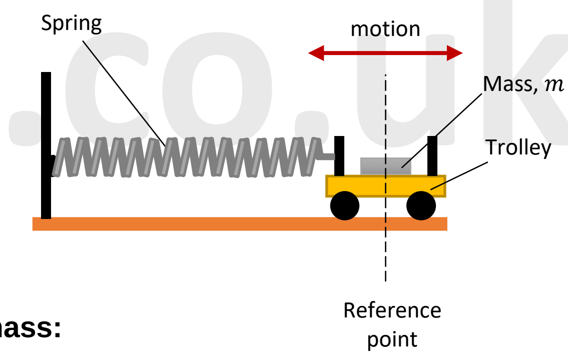
Investigating the mass-spring system

Carrying out the experiment shown below will confirm the relationship shown by the formula $T = 2\pi\sqrt{\frac{m}{k}}$.

To do this experiment, simply change one variable at a time and observe the results.

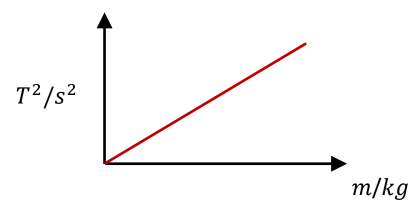
Attach a trolley to a spring, pull it to one side by a specific amount, and then release it. As the spring pulls and pushes the trolley in each direction, the trolley will oscillate.

You can measure the time period, T, by timing how long it takes the mass to complete one oscillation using a reference point and a stopwatch. It is better to measure the time taken to complete 10 oscillations, then divide by the number of oscillations to get an average, as this will reduce random error in your result.



Change the mass:

Change the mass, m, by loading the trolley with masses – remember to include the trolley's mass in your calculations. Since $T \propto \sqrt{m}$, the square of the period, T^2, should be proportional to the mass.

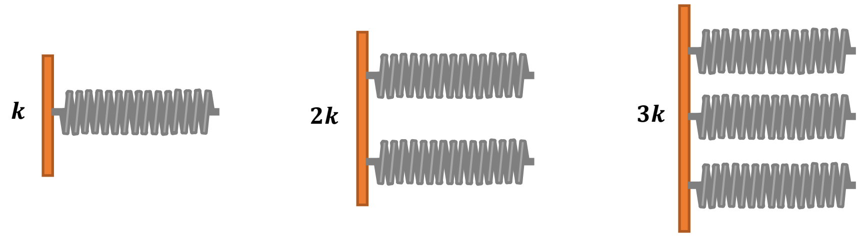


Investigating the mass-spring system

Change the spring constant:

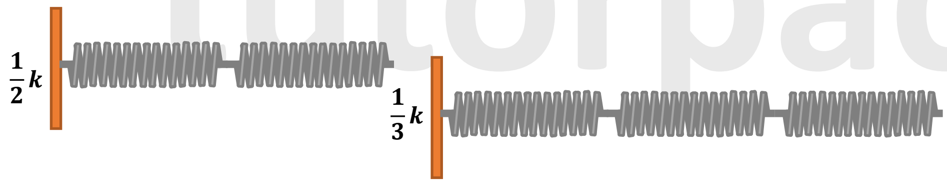
The spring constants add when springs are placed in parallel (side by side):

$$k_{total} = k_1 + k_2 + k_3 \dots \dots \dots$$



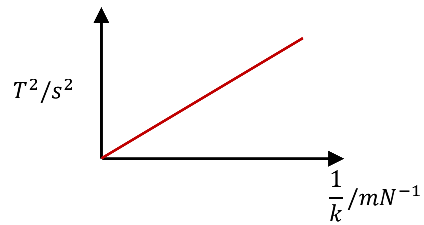
When springs are connected in series (end-to-end), the inverse of the spring constants are added:

$$\frac{1}{k_{total}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \cdots$$



You can change the spring constant, k, in this experiment by using different combinations of springs.

Since $T \propto \sqrt{\frac{1}{k}}$, the square of the period, T^2, should be proportional to the inverse of the spring constant.



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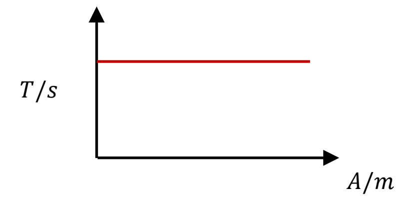
Investigating the mass-spring system

Change the amplitude:

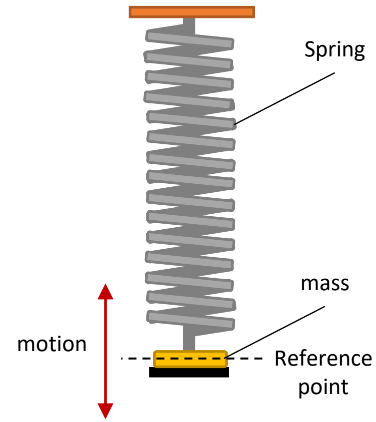
By pulling the trolley by different amounts, you can change the amplitude, A. There should be no change in the period because T is independent of amplitude, A.



What we can improve:

To improve the accuracy of the time period, connect a data logger to a position sensor and use a computer to plot a displacement-time graph.

This experiment can also be done using a vertical mass-spring setup.



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Investigating the Simple Pendulum

We've talked about the simple pendulum throughout this pack, and it's a classic example of an object moving in SHM. The period of an oscillating pendulum is calculated using a formula similar to that of a mass spring system.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

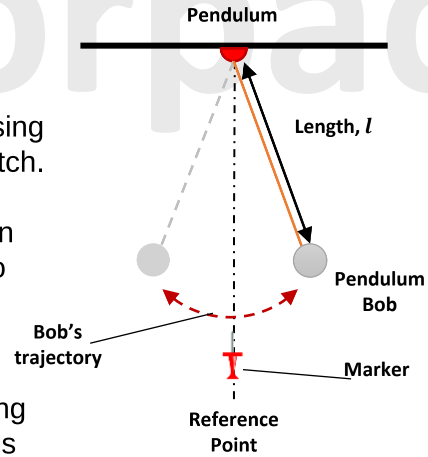
Where:

T = time period in s

l = length of pendulum string in m

g = gravitational field strength, $9.81\ Nkg^{-1}$

Pull a pendulum to one side by a specific amount, then release it. The pendulum is going to swing back and forth.



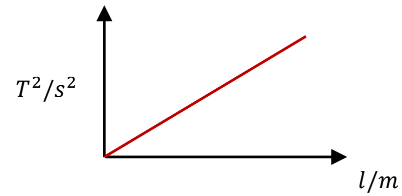
You can measure the period, T, by using a reference point and using a stopwatch. It's a good idea to measure the time taken to complete 10 oscillations, then divide by the number of oscillations to get an average, as this will reduce random error in your result.

To ensure that you start and end timing at the same position in the pendulum's swing, use a marker.
Again, change one variable at a time (e.g. length of pendulum, etc…) and see what happens.

Investigating the Simple Pendulum

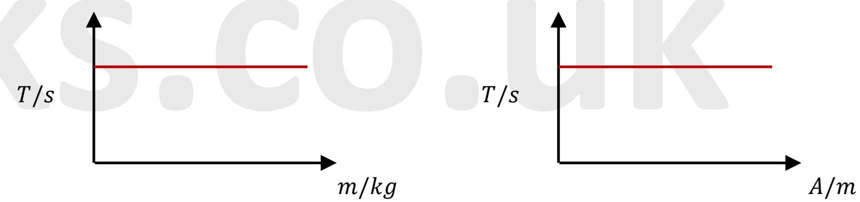
Change the length:

Since the period, T, is proportional to the square root of the length of the pendulum, $\sqrt{l}$, varying l should show that $T^2 \propto l$.



Change mass or amplitude:

T is unaffected by the bob's mass, m, or the oscillation's amplitude, A, therefore altering these should have no effect on T.



What we can improve:

An angle sensor and a computer can be used to reduce random error. The computer will generate a displacement-time graph, from which you can obtain the period, T. To minimise the percentage error in your measurements, make sure to determine the average period over several oscillations.

Please see '14.1.2 Simple Harmonic Motion worked examples' pack for exam style questions.

For more revision notes, tutorials and worked examples please visit www.tutorpacks.co.uk.

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