# A2 Level Physics

Chapter 15 – Gravitational Fields

15.1.1 Gravitation and Planetary Motion

Notes

# Gravitational Fields

The mass of an object produces a gravitational field around it, and this force field exerts an attractive force on any other mass that enters the field region. A gravitational field surrounds all masses, from the tiniest particles of matter to the largest stars.

When an object is dropped, the Earth and the object exert equal and oppositely directed forces on each other; however, because the object's mass is tiny in comparison to the Earth's, the object is drawn towards the Earth.

A Gravitational Field is a region in space in which any mass will experience a force of attraction.

A gravitational field is a force field.

A force field is a region in which a body experiences a non-contact force.

Force fields are created by interactions between objects or particles e.g. between masses in the case of gravity.

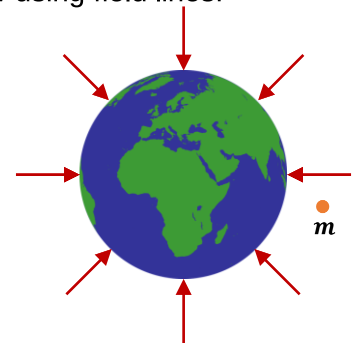
All masses have a gravitational field around them.

Only massive mass objects, such as stars and planets, have gravitational fields that have a noticeable effect. Smaller objects do have gravitational fields that attract other masses, but they are too weak to be detected without specialised equipment.



# Gravitational Field Lines

Gravitational field lines, also known as 'lines of force,' are vectors that show the direction of the force that masses in a gravitational field would experience. The gravitational field of the Earth is shown in the diagram below using field lines:




**Note:**

- If you place a little mass, $m$, anywhere in the Earth's gravitational field, it will always be attracted towards the Earth.
- The Earth's gravitational field is radial meaning that the lines of force meet at the Earth's centre.
- The separation of the field lines in a radial field increases with distance from the centre, indicating that the field strength decreases as distance increases. As a result, the further away you move mass, $m$ from the Earth, the less force it experiences.
- Higher line density shows a stronger gravitational field.

The field is (almost) uniform near the Earth's surface, with field lines that are (almost) parallel and equally spaced. This means that close to the Earth's surface you have a constant gravitational strength and direction.

# Gravitational Field Strength $(g)$

The Field Strength $(g)$ at a point in a gravitational field is the Force $(F)$ per unit mass $(m)$ experienced by a small test mass placed at the point.

In other words, the Earth's gravitational field strength $(g)$ is $9.81\ N/kg$. This means that an object will experience $9.81\ N$ of force for every $kg$ of mass.

To avoid causing a large change in the gravitational field being measured, the test mass must be small.

Field strength $(g)$ is expressed mathematically as:

$$g = \frac{F}{m}$$

Where:

$g$ = gravitational field strength in $Nkg^{-1}$
$F$ = force in $N$
$m$ = mass in $kg$

Note:

The value of $g$ fluctuates slightly depending on where you are on the Earth's surface because of:

- Non-uniformities in the shape and composition of the Earth.
- The Earth's spin reduces $g$ by an amount that varies from zero at the poles to a maximum at the equator.



# Gravitational Field Strength $(g)$

The weight of an object is the force of gravity acting on it. If an object of mass $(m)$ is in a gravitational field of strength $(g)$, the gravitational force $(F)$ on the object is:

$$F = mg$$

If the object is free to fall under the action of this force, it accelerates with an acceleration:

$$a = \frac{F}{m} = \frac{mg}{m} = g$$

This means that, the Field strength at any point in a gravitational field $(Nkg^{-1})$ equals the acceleration of free fall experienced by an object at that point $(ms^{-2})$.

As a result, the object falls freely with an acceleration equal to $g$. So, $g$ can alternatively be described as the acceleration of free fall of an object.

The average gravitational field strength of the Earth's is $9.81\ N\ kg^{-1}$.

Gravitational field strength is a vector quantity.

**Worked example:**

On the Moon, gravity exerts a force of 113.75N on an 70.0kg astronaut. What is the value of $g$ on the Moon?

Just put the numbers into the formula:

$$g = \frac{F}{m} = \frac{113.75N}{70kg} = 1.625 = 1.63Nkg^{-1}(to\ 3s.f.)$$

## Fields

A field, as we know, is a region of space where forces are exerted on objects with certain properties. A gravitational field is only one type of field that can produce a force. The other two are:

1. Electric fields affect anything that has a charge.
2. Magnetic field affects permanent magnets and electric currents.

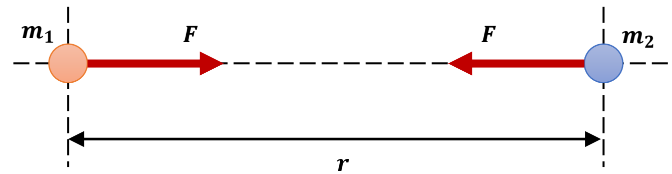
Many of the features of these three types of fields are similar, but there are a few key differences.

In coming packs, we'll go over electric and magnetic fields in more detail.



## Newton's Law of Gravitation

In a gravitational field, the force experienced by an object is always attractive. Every particle in the universe attracts every other particle, although the strength of attraction varies significantly depending on the masses involved and the distance between them.




Consider two point masses ($m_1$ and $m_2$) with distance ($r$) between their centres. The gravitational attraction force ($F$) exerted by each mass on the other can then be calculated using Newton's law of gravitation:

$$F \propto \frac{m_1\, m_2}{r^2}$$

This shows that particles with a force, $F$ is directly proportional to the product of their masses and inversely proportional to the square of their separation.

By inserting a constant of proportionality, we can express Newton's Law of Gravitation mathematically:

$$F = \frac{G\, m_1\, m_2}{r^2}$$

Where:

$F$ = magnitude of force in $N$

$G$ = gravitational constant in $Nm^2kg^{-2}$ $(6.67 \times 10^{-11} Nm^2kg^{-2})$

$m_1$ = mass of the first object in $kg$

$m_2$ = mass of the second object in $kg$

$r$ = distance between the two masses in $m$

# Newton's Law of Gravitation

According to Newton's Law of Gravitation, the gravitational force between any two point objects is:

- Always an attractive force.
- Proportional to the mass of each object.
- Inversely proportional to their separation $(r)$.

Note:

- Newton's law is based on the concept of point masses. The law can be applied to real bodies by assuming that all of a body's mass is concentrated at its centre of mass (e.g. uniform spheres). The separation $(r)$ is the distance between the centre of mass.
- You might also see this formula with a minus sign because the force is always an attractive force. This shows the force's vector nature. But, the minus sign may be omitted because it is generally easier to consider only the magnitude of the force.
- $G$ should not be confused with $g$. $G$ is the gravitational constant $(6.67 \times 10^{-11} Nm^2kg^{-2})$ where as $g$ is the gravitational field strength ($g = 9.81\ ms^{-2}$ on Earth).
- Gravitational forces are extremely weak, unless at least one of the objects is of planetary mass or larger.
- Gravitational forces act at a distance, without the need for an intervening medium.



# Newton's Law of Gravitation

## Inverse Square Law

The Newton's law of gravitation is an example of an inverse square law $\left(F \propto \frac{1}{r^2}\right)$ because it is radial.

This means that when the distance $r$ between the masses increases, the force $F$ decreases. Since it's $r^2$ and not just $r$, if the distance doubles then the force will be one quarter the strength of the initial force, for example:

| **Distance apart** | $r$ | $2r$ | $3r$ | $4r$ |
|---|---|---|---|---|
| Gravitational force | $F$ | $\frac{F}{4}$ | $\frac{F}{9}$ | $\frac{F}{16}$ |

**Worked Example:**

The gravitational force between two objects 10m apart (to 2s.f.) is 0.3N. What will the gravitational force between them be if they move to 35m apart?

Here we use the inverse square law: $F \propto \frac{1}{r^2}$

Therefore $F = \frac{k}{r^2}$, where $k$ is a constant

Find the constant $k = F \times r^2 = 0.3N \times (10)^2 = 30$

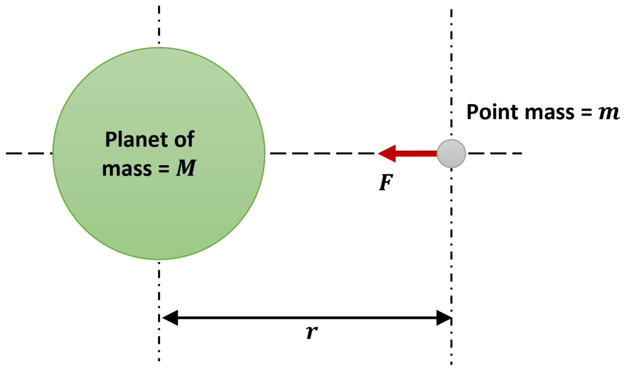
So $F = \frac{30}{r^2}$ so if the two objects now move 35m part the gravitational force between them will be:

$$F = \frac{30}{(35)^2} = 0.024489 \ldots N$$

$$F = 2.4 \times 10^{-2} N \text{ (to 2s.f)}$$

## Gravitational Field Strength of a Point Mass

Consider a point mass $(m)$ located at a distance $(r)$ from the centre of a planet or star with mass $(M)$ and a gravitational field strength $(g)$.




Using the definition of field strength, the force $(F)$ acting on the point mass $(m)$ is:

$$F = mg \dots\dots\dots\dots (1)$$

And applying Newton's law of gravitation, the force $(F)$ is:

$$F = \frac{GMm}{r^2} \dots\dots\dots\dots\dots\dots (2)$$

When you combine equations (1) and (2), you get:

$$mg = \frac{GMm}{r^2}$$

Therefore:

$$g = \frac{GM}{r^2}$$

Where:

$g$ = magnitude of gravitational field strength in $Nkg^{-1}$

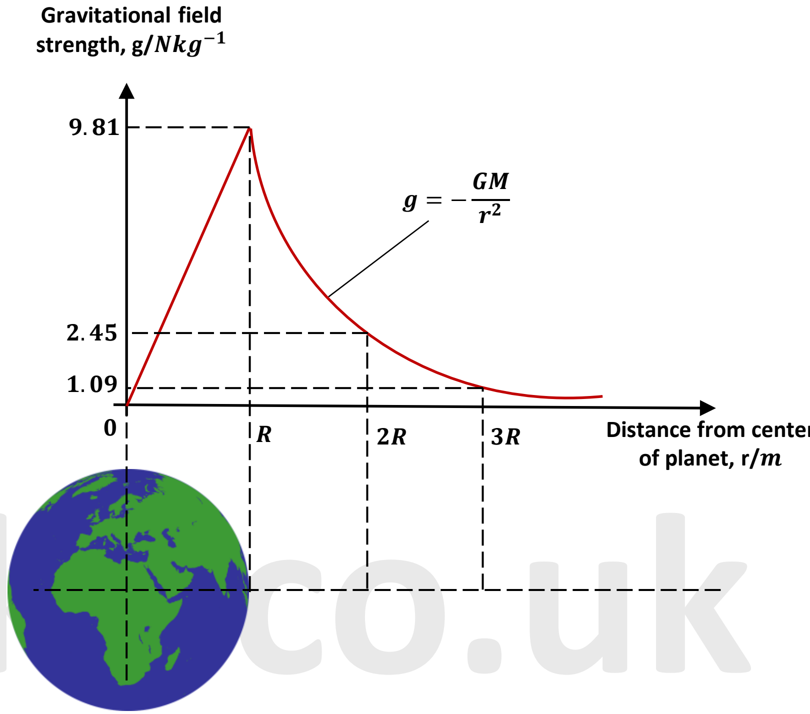
$G$ = gravitational constant in $Nm^2kg^{-2}$

$M$ = mass of the object creating the gravitational field in $kg$

$r$ = distance from the point mass in $m$



## Gravitational Field Strength of a Point Mass




The relationship between gravitational field strength (g) and distance from the Earth's centre (r) is shown in the graph above. From the graph you can see that:

- At the centre of the Earth: $g = 0$.
- Below the surface: $g$ is directly proportional to $r$.
- For $r > R$ (Earth radius): $g$ is inversely proportional to $r^2$.

The gravitational potential can be calculated using the area under the curve.

Note: All of the above can be applied to any planet or star.

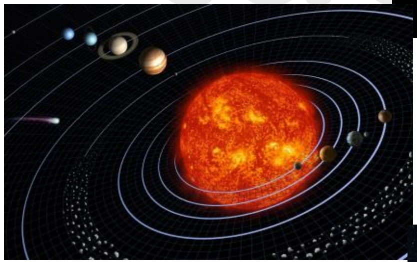
The magnitude of gravitational field strength also obeys the inverse square law as $g \propto \frac{1}{r^2}$.

# Orbits

Planets and satellites in space are kept in orbit by gravitational forces.

A satellite is any body with a small mass orbiting a much larger mass, such as the Moon orbiting the Earth, the planets orbiting the Sun, and so on. These are examples of natural satellites.

Artificial satellites are created by humans and sent into orbit. Due to the gravitational force present between themselves and the Earth, they are able to maintain their orbit at sufficient heights to escape atmospheric friction, which would dissipate their energy and bring them crashing back to Earth.

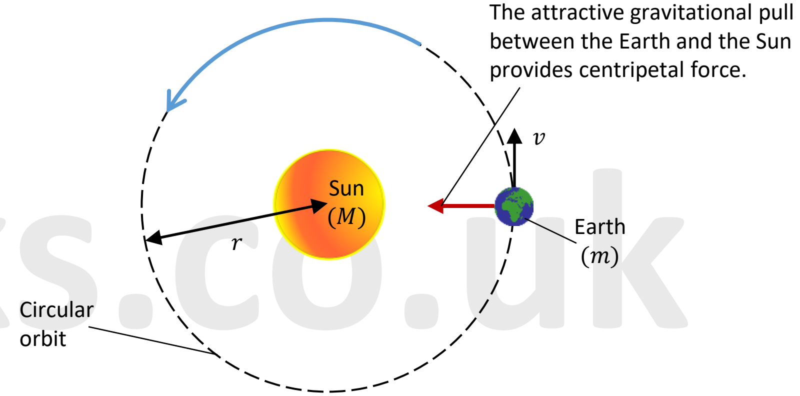




# Orbits

Satellites are held in orbit by gravitational 'pull' of the mass they're orbiting. Since the planets in our solar system have almost circular orbits, you can study their orbital speed and period using circular motion equations.

A centripetal force keeps any object undergoing circular motion (such as a satellite) in its trajectory. What causes this force depends on the object. In the case of satellites, it's the gravitational attraction of the mass they're orbiting. Therefore, the gravitational force is the centripetal force in this case.




In the diagram above, the Earth of mass $(m)$ orbits the Sun of mass $(M)$ at a speed $(v)$ and an orbital radius $(r)$. The gravitational force acting between the Sun and the Earth provides the centripetal force required for circular motion. Therefore:

Gravitational force = centripetal force

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Therefore: $v^2 = \frac{G\,M\,\cancel{m}\,\cancel{r}}{r^{\cancel{2}}\,\cancel{m}}$

$$v^2 = \frac{GM}{r} \ldots\ldots\ldots\ldots (1)$$

## Orbits

Therefore, the orbital speed of a satellite is:

$$v = \sqrt{\frac{GM}{r}}$$

So, a satellite's orbital speed is inversely proportional to the square root of its orbital radius, or:

$$v \propto \frac{1}{\sqrt{r}}$$

The orbital period, $T$, is the amount of time it takes for a satellite to complete one orbit. To calculate the orbital period re-call,

$$speed = \frac{distance\ travelled\ in\ one\ complete\ orbit}{time\ taken\ for\ one\ complete\ orbit\ (Period)}$$

$$v = \frac{2\pi r}{T}$$

Then substituting for $v$ in equation $(1)$ gives:

$$\frac{(2\pi r)^2}{T^2} = \frac{GM}{r}$$

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

Therefore:

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

So the orbital period is:

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

This shows that the orbital period squared is proportional to the orbital radius cubed:

$T^2 \propto r^3$ or $T \propto \sqrt{r^3}$



## Orbits

This means that the larger the orbital radius of a satellite, the slower it will travel and the longer it will take to complete one orbit.

The equation below shows that for a given planet or star, the ratio $\left(\frac{T^2}{r^3}\right)$ is a constant for all of its satellites, regardless of the planets mass.

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM} \dots \dots \dots \dots (2)$$

To prove the above, Newton had assumed:

- The planets and the Sun were all point masses.
- The gravitational force between the Sun and the planets was directly proportional to their masses and inversely proportional to the square of their distance apart.

### Kelper's 3rd Law

Forty years or so earlier, Johannes Kepler, an astronomer, had made careful observations and came up with Kepler's 3rd law, which related two quantities:

1) The average distance of a planet from the centre of the Sun, $r$.
2) The length of time it takes for the planet to complete one orbit of the Sun, its period, $T$.

Using his measurements, he discovered that:

The period squared is proportional to the mean radius cubed $T^2 \propto r^3$

As a result, Kepler proposed his 3rd Law of Planetary Motion which stated:

The ratio $\left(\frac{T^2}{r^3}\right)$ is the same (i.e. constant) for all planets.

Newton was then able to use his Theory of Gravitation to prove Kepler's third law.

# Orbits

We can rearrange equation (2) to give the following:

$$M = \frac{4\pi^2 r^3}{GT^2}$$

This equation allows us to calculate the Mass $(M)$ of the central planet or star from the Period $(T)$ and orbit radius $(r)$ of one its satellites.

**Worked Example:**

Planets A and B are orbiting the same star. Planet A has a period of 19.5 hours and an orbital radius of $7.0 \times 10^{10} m$. The orbital radius of Planet B is $1.5 \times 10^{12} m$. Calculate the orbital period of planet B in hours.

$T^2 \propto r^3$, so $\frac{T^2}{r^3} = constant$

Therefore: $\frac{T_A^2}{r_A^3} = \frac{T_B^2}{r_B^3}$ and so $T_B^2 = \frac{T_A^2 r_B^3}{r_A^3}$

$$T_B = \sqrt{\frac{T_A^2 r_B^3}{r_A^3}} = \sqrt{\frac{(19.5)^2 \times (1.5 \times 10^{12})^3}{(7.0 \times 10^{10})^3}}$$

$$\therefore\ T_B = 1934\ hours\ (to\ 2s.f.)$$

# Orbits

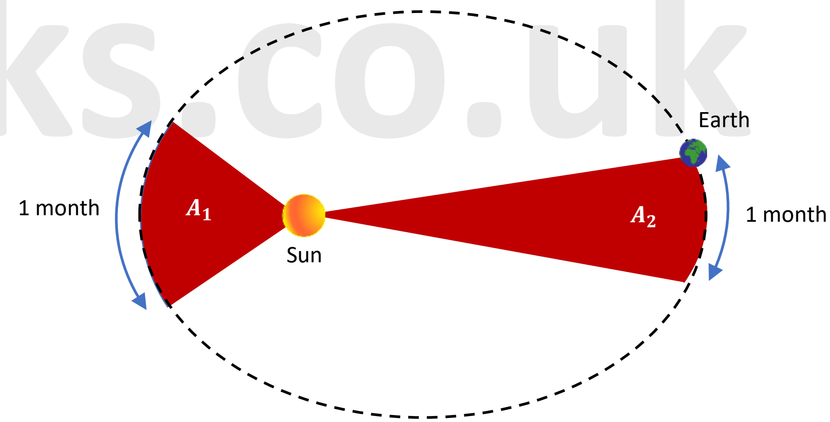
## Kepler's 1st and 2nd Law

### Kepler's 1st Law

According to Kepler's first law, all planets move about the Sun in elliptical orbits, with the Sun at one focus. Since the ellipse's curvature is so low, the motion can be modelled as circular.

### Kepler's 2nd Law

According to Kepler's second law, a line segment connecting a planet and the Sun sweeps out equal areas at equal time intervals. This is because the planet's speed is not constant; it moves faster as it gets closer to the Sun.




$A_1 = A_2$

## Orbits

Newton wondered how the Moon knows Earth is there, proposing that gravity maintains its orbit. Though scientists haven't fully understood this concept, Newton's formula proved valuable to NASA for planning the six Apollo Moon missions four decades ago.

Even before space travel, data on the Moon's orbit allowed calculations of Earth's mass. With the Moon's orbit period of 27.3 days ($T = 2.35 \times 10^6\ s$) and average radius of $384\ 000\ km$, ($r = 3.84 \times 10^8\ m$), Earth's mass can be determined.

Gravity is the cause of the centripetal force, so:

Gravitational force = centripetal force

$$\frac{G m_E m_M}{r^2} = \frac{m_M v^2}{r}$$

Simplifying gives:

$$m_E = \frac{r v^2}{G}$$

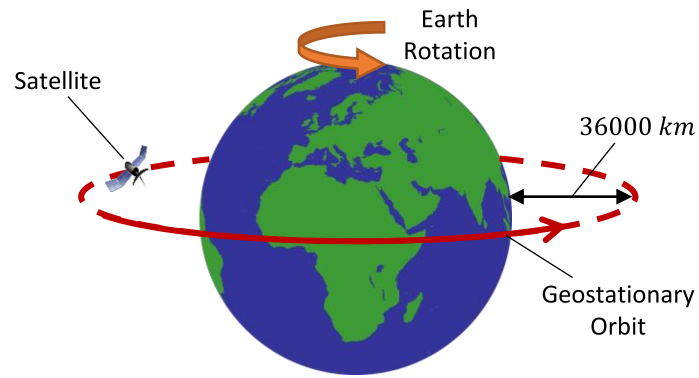
The speed of the Moon comes from the time it takes to orbit:

$$v = \frac{2\pi r}{T} = \frac{2\pi \times 3.84 \times 10^8}{2.36 \times 10^6} = 1020\ m\ s^{-1}$$

$$\therefore m_E = \frac{r v^2}{G} = \frac{(3.84 \times 10^8)(1020^2)}{6.67 \times 10^{-11}}$$

$$m_E = 6.02 \times 10^{24}\ kg$$

## Geostationary Orbits




A Geostationary satellite is in a geostationary orbit. This means that it:

- Travels above the equator in the same direction as that of the Earth (west to east).
- Has the same orbital period as the Earth's rotation around its own axis (24 hours).
- Always appears to be above the same point on the Earth's surface.

This orbit allows the satellite to continuously monitor the same area of the Earth.

Geostationary satellites are useful for transmitting TV and phone signals because they are stationary relative to the Earth's surface, so you don't have to adjust the angle of your receiver (or transmitter) to keep up.

# Geostationary Orbit

**Worked example:**

Calculate the height above the Earth that a satellite must be placed for it to orbit in a geostationary manner.

(mass of Earth $= 6.0 \times 10^{24}\ kg$, radius of Earth $= 6.4 \times 10^{6}\ m$)

**Answer:**

The gravitational force acting between the Earth and the satellite provides the centripetal force required for circular motion and to keep the satellite in orbit, therefore:

$$F = \frac{GMm}{r^2} \text{ where } F = centripetal\ force = ma = m\omega^2 r.$$

$$\therefore m\omega^2 r = \frac{GMm}{r^2}$$

Simplifying gives us:

$$\omega = \sqrt{\frac{GM}{r^3}}$$

The angular speed of the satellite is:

$$\omega = \frac{2\pi}{T}$$

And the time period required for a geostationary orbit is $24\ h = 86{,}400\ s$

Therefore: $\frac{2\pi}{T} = \sqrt{\frac{GM}{r^3}}$

$$r^3 = \frac{GMT^2}{(2\pi)^2} = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 86400^2}{4\pi^2} = 7.57 \times 10^{22}$$

$$r = \sqrt[3]{7.57 \times 10^{22}} = 4.23 \times 10^7 m$$

This is the radius of the satellite's orbit. The radius of the Earth is $6.4 \times 10^6\ m$, so the height of the satellite above the Earth's surface is:

$$42.3 \times 10^6 - 6.4 \times 10^6 = 3.59 \times 10^7\ m$$

So, the satellite is approx. $3.6 \times 10^7\ m$ above the Earth's surface.



# Low Orbiting Satellites

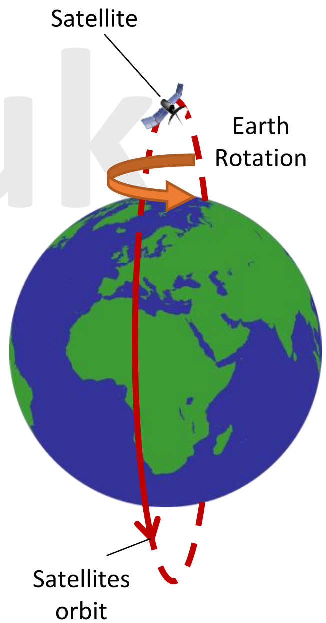
Low-orbiting satellites are those that orbit between 180 and 2000 kilometres above the Earth's surface. These satellites are cheaper to launch and require less powerful transmitters as the satellites are closer to the Earth. Low-orbiting satellites are great for communications.

However, because of their close proximity to Earth and fast orbital speed (in comparison to Earth's), several satellites must work together to provide continuous coverage.

Satellites in low orbit are close enough to see the Earth's surface in detail. Imagining satellites are typically positioned in this orbit and are used for imaging (e.g. mapping and spying) as well as weather monitoring.

Their orbits are normally aligned to include the north and south poles. These satellites do not stay over the same section of the Earth since the planet and the satellite rotate at different angular speeds, allowing the whole surface to be scanned.

The international Space Station is an example of a low orbiting satellite.



# Gravitational Potential

You might remember from GCSE that the increase in gravitational potential energy of a body when it is raised through a height $\Delta h$ is determined by the formula:

$$E_P = mg\Delta h$$

This formula, however, only works on Earth; what about gravitational potential energy on an astronomical scale? At astronomical scale it is the Gravitational potential, and this is not to be confused with the gravitational potential energy.

**What is the gravitational potential?**

Every object in a gravitational field has a gravitational potential that increases as they move away from the centre of the field.

The gravitational potential, $V$, at a point is the gravitational potential energy that a unit mass at that point would have or the amount of work done per unit mass to move an object from an infinite distance to that point in the field.

For example, if a $1\ kg$ mass has $-10\ J$ of potential energy at a point Z, the gravitational potential at $Z$ is $-10\ Jkg^{-1}$. The equation for gravitational potential in a radial field (such as the Earth's) is:

$$V = -\frac{GM}{r}$$

Where:

$V$ = gravitational potential in $Jkg^{-1}$

$G$ = gravitational constant in $Nm^2kg^{-2}$

$M$ = mass of the object causing the gravitational field in $kg$

$r$ = distance from the centre of the object in $m$

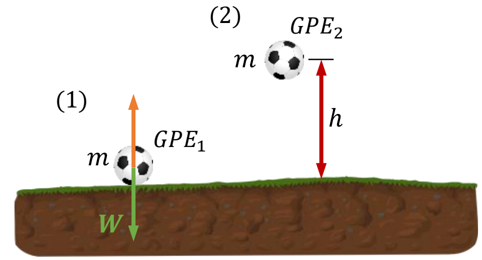




## Gravitational Potential

Let's take a closer look at this formula and reflect on our GCSE knowledge, where it was taught that an object with a mass of $m$ on the surface of the Earth has zero gravitational potential energy.

But, if we raise the object to a height $h$, it will gain gravitational potential energy.




Although it is commonly said that an object on the surface of the Earth has zero gravitational potential energy (GPE), it is not entirely true. This simplification is made to help understanding in certain contexts, such as in the case of a pendulum that has zero GPE at the bottom of its swing. However, cutting the string of the pendulum would cause the object to fall.

An object on the surface of the Earth has gravitational potential energy (GPE) acting downwards, even though it is not falling. The reason for this is that the surface of the Earth provides an equal and opposite force, cancelling out the downwards force and preventing the object from falling. If the surface were to disappear, the object would fall. Thus, the object has GPE, which can be referred to as $GPE_1$.

If we lift this mass, we'll have to put in some work, and it'll have a new GPE at this height. We can call that $GPE_2$. To find out the gravitational potential energy at that height we can just use:

$$GPE_2 = mgh$$

However, the object on the Earth's surface has a GPE that is not equal to zero, so we would examine the change in the GPE from point (1) to point (2). As a result, the equation above becomes:

$$\Delta GPE = mgh$$

This equation we learned in GCSE physics shows that as an object is lifted, its gravitational potential energy (GPE) increases. This means that a change occurs in the object's GPE.

But, a change in potential energy is also know as work done.

## Gravitational Potential

We know that Work done is:

$$Work\ done = Force \times distance$$
$$W = F \times d$$

We also know that $F = mg$

So:

$$W = F\,d$$

$$\Delta GPE = mg\,h$$

This shows the link between $GPE$ and $W$. The force is $mg$ in $GPE$, and the height, $h$ from $GPE$ is distance $d$ in $W$. $W$ and $GPE$ are both measured in Joules.

To lift an object to a certain height, work must be done, and energy must be put into the system. Due to the principle of energy conservation, the energy must go somewhere, and it goes into the new gravitational potential energy ($GPE$) from the work done. Neglecting resistive forces, the energy put in becomes the $GPE$ of the object at its new height, $h$.

However, GPE has limitations because it only takes into account g, which is the strength of gravity on Earth ($9.81\ ms^{-1}$). If $h$ is small, $\Delta GPE = mgh$ is a fantastic equation to calculate gravitational potential energy, and it works well for objects on Earth. But what if I want to calculate the gravitational potential energy of objects in Earth's orbit? Because the gravitational field strength decreases as the object goes further away from the Earth, $g = 9.81ms^{-1}$ no longer holds true at those points. As a result, the value of $g$ will vary and will no longer be constant.

But $\Delta GPE = mgh$ doesn't consider changing $g$ values; instead, its meant to have a constant $g$ value.

So we can use $g = \frac{GM}{r^2}$ to find the new value of $g$ and plug it back into $\Delta GPE = mgh$. However, this would only be true if $g$ remained constant above a constant distance $h$ from the Earth's centre. But, what if, the object was always moving away from the Earth? For each new position, we can't keep calculating the $g$ value. As a result, we'll need to derive a dynamic equation that allows for $g$ to change.



## Gravitational Potential

So:

$$\Delta GPE = W = F \times d$$

$F = mg$ and $g = \frac{GM}{r^2}$. Therefore combining the two equations gives us:

$$F = mg = \frac{mGM}{r^2}$$

As a result:

$$\Delta GPE = F \times d = \frac{GMm}{r^2} \times r$$

I selected $r$ because it represents the distance from the Earth's centre.

Then simplifying this we get:

$$\Delta GPE = \frac{GMm}{r}$$

This is a new equation that calculates the GPE for an object of mass $m$, regardless of how far away it is from Earth. Furthermore, it cancels out $g$.

However, because this equation relies on $m$, the value will vary depending on the object. But we want a generic equation, one that applies to any object, regardless of mass.

Luckily, by moving the $m$ to the other side, we may turn it into a general equation:

$$\frac{\Delta GPE}{m} = \frac{GM}{r}$$

This simply means that you have the change of gravitational potential energy per unit mass $\left(\frac{\Delta GPE}{m}\right)$. This would be the general equation. However, the change in gravitational potential energy per unit mass, will require its own letter, which is $V$. Therefore:

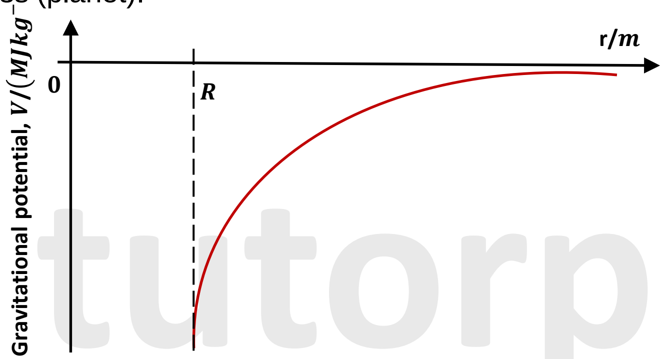
$$V = \frac{GM}{r}$$

# Gravitational Potential

$V$ stands for gravitational potential, which is defined as the work done per unit mass to move a unit mass from infinity to that point.

The field is negligible at infinity, so $V$ is $0$, which is its maximum value. $V$ is negative at all other points, indicating how much energy is necessary to move the object out of the field.

As seen below, the gravitational potential can be plotted against the distance from the centre of a mass (e.g., planet). $R$ is the radius of the mass (planet).




If you find the gradient of this graph at a particular point, you get the value of $-g$ at that point. So:

$$g = -\frac{\Delta V}{\Delta r}$$

Where:

$g$ = gravitational field strength in $Nkg^{-1}$

$\Delta V$ = change in gravitational potential in $Jkg^{-1}$

$\Delta r$ = change in distance from the centre of the object in $m$



# Gravitational Potential

Note:

- This isn't the inverse square law. Rather, the gravitational potential halves when the distance is doubled. When the distance triples the potential decreases by a third.

- $V$ can also be represented as:

$$V = \frac{-GM}{r}$$

The negative sign just implies that gravity is an attractive force.

Remember you can find the area under a $g - r$ graph to give you $\Delta V$, the change in gravitational potential between two radial distances.

# Gravitational Potential

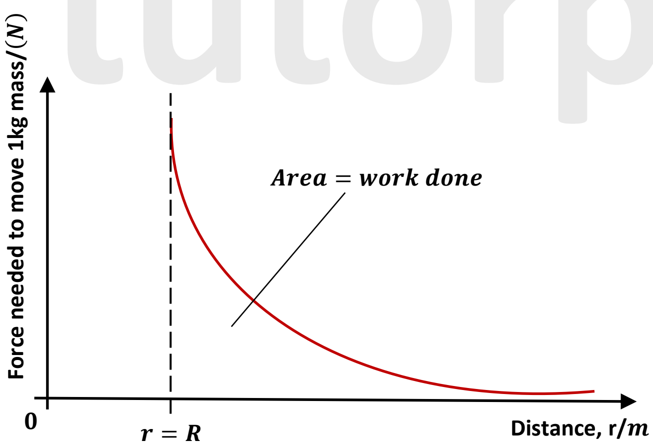
The change in gravitational potential, $\Delta V$, is calculated by subtracting the potential at one distance from the potential at the other:

$$\Delta V = -GM\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Since $V$ is the energy per unit mass, you can multiply the above equation by the mass of the object $(m)$ to get the energy (or work done) required to go from potential $V_1$ to $V_2$. Therefore, the total energy to complete this movement is:

$$\Delta GPE = \Delta E_P = m\Delta V$$

This equation can also be shown graphically. The work done is the area under a force-distance graph.

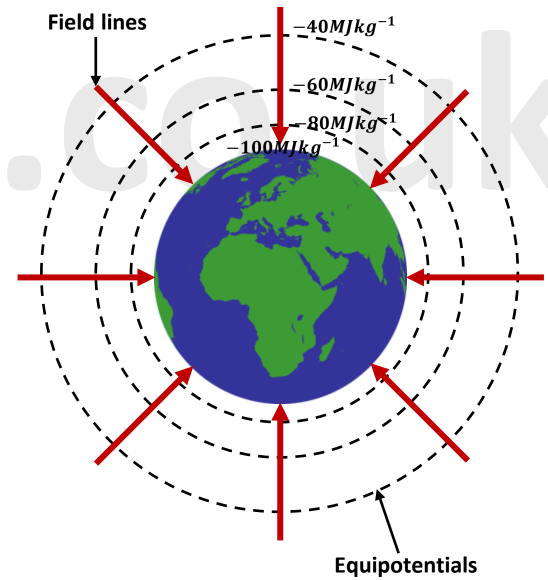





# Equipotentials

Equipotentials are surfaces (in 3D) and lines (in 2D) that connect all points with the same gravitational potential, $V$. This means that the potential does not change while you move along an equipotential - you do not lose or gain energy. As a result, the gravitational potential difference is zero as you go along the equipotential: $\Delta V = 0$. As $\Delta W = m\Delta V$, this means that the amount of work done is also zero.

Equipotentials for a uniform spherical mass are spherical surfaces. The field lines and the equipotentials are always perpendicular. The equipotentials and fields lines around Earth are shown in the diagram below. This is a 2D representation of a 3D shape. In reality, the equipotentials would resemble spherical shells that go all the way around the Earth.



## Escape Velocity

The kinetic energy of an object at the start must be equal to or greater than the gravitational potential energy necessary to lift it to infinity in order for it to escape a gravitational field formed by a mass, $M$. Regardless of the mass $m$ of the object, the escape velocity is the same for any object at that beginning radius, $r$.

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

So:

$$v = \sqrt{\frac{2GM}{r}}$$

## Similarities and Difference between Electric and Gravitational Fields

| | Electric Field | Gravitational Field |
|---|---|---|
| Similarities | | |
| Obey the inverse square law of force | Coulomb's law of force $F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$ | Newton's law of gravitation $F = \frac{Gm_1 m_2}{r^2}$ |
| Have Uniform Fields | $E$ is constant field lines are parallel | $g$ is constant field lines are parallel |
| Have Radial fields | Due to a point charge $(Q)$ $E = \frac{Q}{4\pi\varepsilon_0 r^2}$ | Due to a point mass $(M)$ $g = -\frac{GM}{r^2}$ |
| Differences | | |
| Action | Between any two charged objects | Between any two masses |
| Type of Force | Unlike charges attract<br>Like charges repel | Attraction only |
| Constant of proportionality | $\frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 Nm^2C^{-2}$ | $G = 6.67 \times 10^{-11} Nm^2kg^{-2}$ |

Please see **'15.1.2 Gravitation and Planetary Motion worked examples'** pack for exam style questions.

For more revision notes, tutorials and worked examples please visit **www.tutorpacks.co.uk.**