AS Level Physics

Chapter 6 – Further Mechanics

6.2.1 Collisions and 2D Momentum

Notes

Content

MOMENTUM AND COLLISIONS

Momentum:

The momentum of an object is the product of its mass and velocity.

$$\rho = m \times v$$

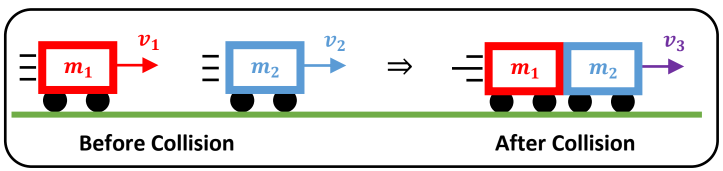
where:
ρ = Momentum measured in $kg\ ms^{-1}$.
m = mass measured in kg.
v = velocity measured in ms^{-1}.

Momentum is a vector and therefore has both magnitude and direction. This means momentum to the right can be considered positive and momentum to the left can be negative.

THE PRINCIPLE OF CONSERVATION OF MOMENTUM

"The total momentum is conserved in collisions provided there are no external forces (e.g. friction)"

This means that in a collision total momentum before the collision is equal to the total momentum after the collision.



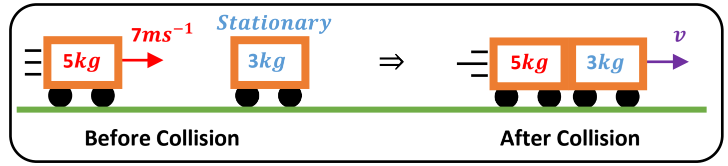
Total momentum before collision = **Total momentum after collision**

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v_3$$

MOMENTUM AND COLLISIONS

Worked Example 1:

A 5kg mass travelling at 7 ms^{-1} collides with a stationary 3kg mass and sticks to it. Calculate the velocity just after impact.



Answer:

Total momentum before = Total momentum after

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v_3$$

$$(5kg \times 7ms^{-1}) + (3kg \times 0ms^{-1}) = (5kg + 3kg)v$$

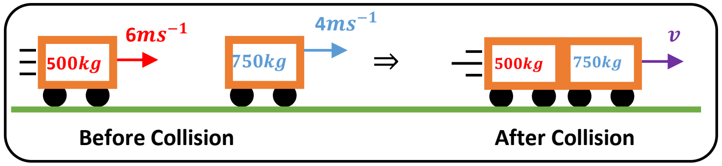
$$35 = 8v$$

$$\boldsymbol{v = 4.4ms^{-1}\ to\ the\ right}$$

MOMENTUM AND COLLISIONS

Worked Example 2:

A car with a mass 500 kg travels at a speed of $6ms^{-1}$. It collides and joins with a 750 kg car travelling at $4ms^{-1}$. Calculate the velocity of the cars just after the impact.



Answer:

Total momentum before = Total momentum after

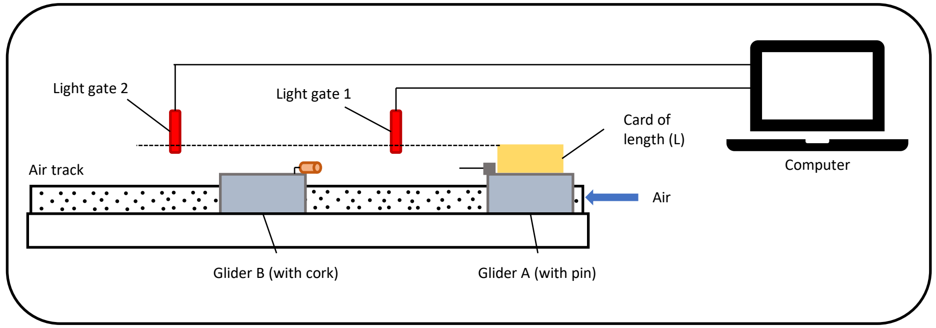
$$m_1v_1 + m_2v_2 = (m_1 + m_2)v_3$$

$$(500kg \times 6ms^{-1}) + (750kg \times 4ms^{-1}) = (500kg + 750kg)v$$

$$6000 = 1200v$$

$$\boldsymbol{v = 5ms^{-1}\ to\ the\ right}$$

EXPERIMENT TO TEST THE CONSERVATION OF MOMENTUM



1) Set up the equipment as shown on the above diagram.

2) Start by measuring the masses of gliders 1 & 2 using an electronic balance.

3) Glider A is pushed towards the stationary Glider B. Using a computer, Glider A's speed (v_1) is calculated when it passes light gate 1 by using the formula below:

$$v_1 = \frac{length\ of\ card\ going\ through\ the\ light-gate}{time\ taken\ for\ card\ to\ pass\ through\ the\ light-gate} = \frac{L}{t}$$

- Glider A then collides with Glider B, piercing the pin into the cork. This results in the two gliders sticking together.
- The two attached gliders then pass through light-gate 2 and their common speed (v_2) is calculated by the computer.
- Make sure to only record the before and after velocities when the gliders collide and the pin pierces the cork forming one mass.
- Repeat the experiment at least 3 more times.

<u>Result:</u> From the experiment you should see the momentum before the collision is equal to the momentum after the collision.

TYPES OF COLLISIONS

In any collision between two objects:

1) Total momentum is always conserved (as long as no external forces act on the system).

2) Total energy is always conserved.

(This is due to the principle of conservation of energy which states that energy cannot be created nor destroyed, it can only be converted from one form into another).

- **However, kinetic energy (KE) can or cannot be conserved.** There are two types of collisions:

1. Elastic Collision:

An elastic collision is where kinetic energy (KE) and momentum is conserved.

i.e. total KE before collision = total KE after collision.

2. Inelastic Collision:

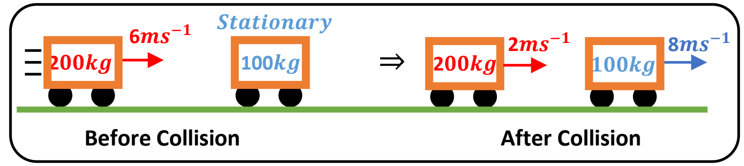
An Inelastic collision is where kinetic energy (KE) is NOT conserved however momentum is conserved.

$$Total\ KE\ after\ collision\ < Total\ KE\ before\ collision$$

TYPES OF COLLISIONS

Elastic Collision Worked Example 1:

A 200kg car travels with a speed of $6ms^{-1}$ and collides with a stationary 100kg vehicle. Following the collision, the 200kg vehicle moves off at $2ms^{-1}$ and the 100kg vehicle at $8ms^{-1}$. Show the collision is elastic.



Answer:

Momentum:

Before:

$$m_1v_1 + m_2v_2$$

$$(200kg \times 6ms^{-1}) + (100kg \times 0ms^{-1})$$

$$Momentum\ Before = 1200kgms^{-1}$$

After:

$$m_1v_3 + m_2v_4$$

$$(200kg \times 2ms^{-1}) + (100kg \times 8ms^{-1})$$

$$Momentum\ After = 1200kgms^{-1}$$

Momentum has been conserved.

Kinetic Energy:

Before:

$$E_K\ total = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\left(\frac{1}{2} \times 200kg \times (6ms^{-1})^2\right) + \left(\frac{1}{2} \times 100kg \times (0ms^{-1})^2\right)$$

$$E_K\ total\ Before = 3600J$$

After:

$$E_K\ total = \frac{1}{2}m_1v_3^2 + \frac{1}{2}m_2v_4^2$$

$$\left(\frac{1}{2} \times 200kg \times (2ms^{-1})^2\right) + \left(\frac{1}{2} \times 100kg \times (8ms^{-1})^2\right)$$

$$E_K\ total\ After = 3600J$$

Kinetic Energy has been conserved.

As momentum and kinetic energy are conserved this is an elastic collision.

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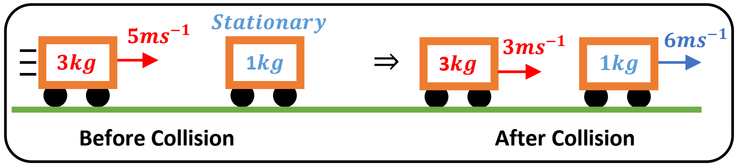
TYPES OF COLLISIONS

Inelastic Collision Worked Example 1:

A 3kg block is travelling at $5ms^{-1}$ when it collides with a stationary 1kg trolley. Then, they move off at $3ms^{-1}$ and $6ms^{-1}$ respectively. Show that this collision is inelastic.



Answer:

Momentum:

Before:

$$m_1v_1 + m_2v_2$$
$$(3kg \times 5ms^{-1}) + (1kg \times 0ms^{-1})$$
$$Momentum\ Before = 15kgms^{-1}$$

After:

$$m_1v_3 + m_2v_4$$
$$(3kg \times 3ms^{-1}) + (1kg \times 6ms^{-1})$$
$$Momentum\ After = 15kgms^{-1}$$

Momentum has been conserved.

Kinetic Energy:

Before:

$$E_K\ total = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$
$$\left(\frac{1}{2} \times 3kg \times (5ms^{-1})^2\right) + \left(\frac{1}{2} \times 1kg \times (0ms^{-1})^2\right)$$
$$E_K\ total\ Before = 37.5J$$

After:

$$E_K\ total = \frac{1}{2}m_1v_3^2 + \frac{1}{2}m_2v_4^2$$
$$\left(\frac{1}{2} \times 3kg \times (3ms^{-1})^2\right) + \left(\frac{1}{2} \times 1kg \times (6ms^{-1})^2\right)$$
$$E_K\ total\ After = 31.5J$$

Kinetic Energy has NOT been conserved.

As momentum is conserved and KE is not conserved this is an inelastic collision.

TYPES OF COLLISIONS

Note:

In reality, most collisions are inelastic because some KE is always converted to heat and sound energy on impact.

Perfectly Inelastic

A perfectly inelastic collision is where all the initial energy is transferred to other energy forms (i.e. heat or sound) therefore:

$$KE\ after\ collsion = 0$$

TYPES OF COLLISIONS

HEAD ON COLLISIONS

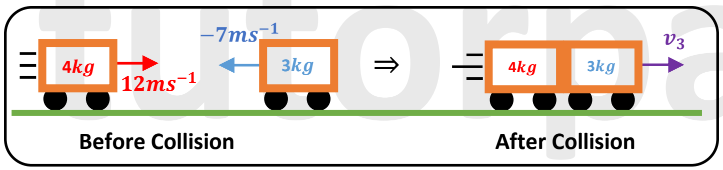
Head on collisions occur when objects travel in opposite directions but towards each other. If one objects direction/velocity is POSITIVE, the other objects direction/velocity has to be NEGATIVE.

Worked Example 1:

A 4kg object moves at $12ms^{-1}$ and collides head-on with a 3kg object travelling at $7ms^{-1}$. After the collision, they both move off together.

a) Calculate the velocity of the objects immediately after they collide.

b) Find out if the collision is elastic or inelastic.



TYPES OF COLLISIONS

Answer:

Momentum:

Momentum is conserved therefore:

Total momentum before = Total momentum after

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v_3$$

$$(4kg \times 12ms^{-1}) + (3kg \times -7ms^{-1}) = (4kg + 3kg)v$$

$$48 - 21 = 7v$$

$$7v = 27$$

$$\boldsymbol{v_3 = 3.86ms^{-1}\ to\ the\ right}$$

Kinetic Energy:

Before:

$$E_K\ total = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\left(\frac{1}{2} \times 4kg \times (12ms^{-1})^2\right) + \left(\frac{1}{2} \times 3kg \times (-7ms^{-1})^2\right)$$

$$E_K\ total\ Before = 361.5J$$

After:

$$E_K\ total = \frac{1}{2}(m_1+m_2)v_3^2$$

$$\left(\frac{1}{2} \times 7kg \times (3.86ms^{-1})^2\right)$$

$$E_K\ total\ After = 52.1J$$

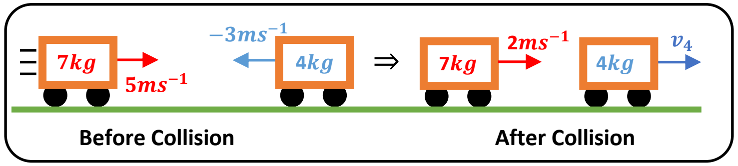
Kinetic Energy has NOT been conserved **therefore this is an inelastic collision.**

TYPES OF COLLISIONS

HEAD ON COLLISIONS

Worked Example 2:

Two objects collide as shown:



a) Calculate the velocity at which the 4kg object moves immediately after impact

b) Find out if the collision is elastic or inelastic

Answer:

Momentum:

Momentum is conserved therefore:

Total momentum before = Total momentum after

$$m_1v_1 + m_2v_2 = m_3v_3 + m_4v_4$$

$$(7kg \times 5ms^{-1}) + (4kg \times -3ms^{-1}) = (7kg \times 2ms^{-1}) + (4kg \times v_4)$$

$$23 - 14 = 4v_4$$

$$4v_4 = 9$$

$$\boldsymbol{v_4 = 2.25ms^{-1}\ to\ the\ right}$$

TYPES OF COLLISIONS

HEAD ON COLLISIONS

Worked Example 2:

Solution: Continued

Kinetic Energy:

Before:

$$E_K\ total = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\left(\frac{1}{2} \times 7kg \times (5ms^{-1})^2\right) + \left(\frac{1}{2} \times 4kg \times (-3ms^{-1})^2\right)$$

$$E_K\ total\ Before = 105.5J$$

After:

$$E_K\ total = \frac{1}{2}m_1v_3^2 + \frac{1}{2}m_2v_4^2$$

$$\left(\frac{1}{2} \times 7kg \times (2ms^{-1})^2\right) + \left(\frac{1}{2} \times 4kg \times (2.25ms^{-1})^2\right)$$

$$E_K\ total\ After = 24.13J$$

KE is not conserved therefore it is an inelastic collision.

Collision vectors

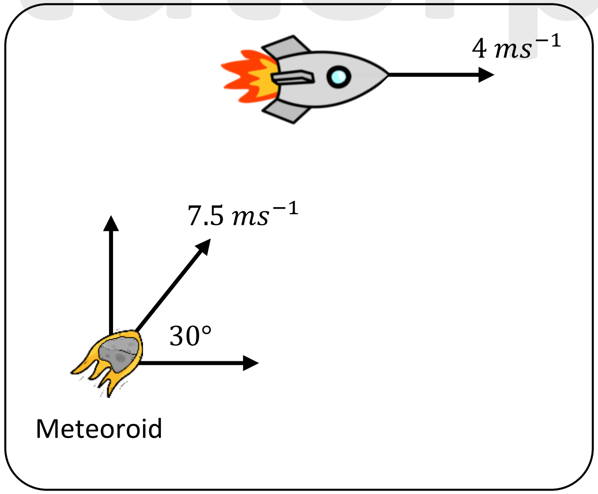
Conservation of linear momentum in two dimensions

Momentum is conserved when no external forces are present in an interaction, ensuring that the momentum before an event, like a collision, equals the momentum after. Additionally, momentum conservation may vary across dimensions.

To solve problems involving the conservation of momentum in two dimensions, break down the motion into components along perpendicular axes (e.g., x and y-axis), then solve the resultant pair in one-dimension simultaneously.

For example:

A spacecraft moves at $4\ ms^{-1}$ in space. A meteoroid, traveling at $7.5\ ms^{-1}$, collides with the spacecraft at a $30°$ angle and becomes embedded into it. The spacecraft has a mass of $320\ kg$ and the meteoroid has a mass of $10\ kg$. Calculate the spacecraft's velocity after the collision.



Collision vectors

Step 1: Calculate the momentum of the meteoroid and rocket in the x-axis:

Find the velocity of the meteoroid in the x-axis:

$$v_{meteoroid} = 7.5\cos(30°) = 6.50\ \text{ms}^{-1}$$

Calculate the momentum of the meteoroid in the x-axis is:

$$\rho_{meteoroid} = 10 \times 6.50 = 65.0\ kg\ ms^{-1}$$

Determine the momentum of the rocket is:

$$\rho_{rocket} = 320 \times 4 = 1280\ kg\ ms^{-1}$$

Calculate the total momentum in the x-axis is:

$$\rho_x = \rho_{meteoroid} + \rho_{rocket}$$
$$\rho_x = 65.0\ kg\ ms^{-1} + 1280\ kg\ ms^{-1} = 1345\ kg\ ms^{-1}$$

Step 2: Calculate the momentum of the meteoroid and rocket in the y-axis:

Find the velocity of the meteoroid in the y-axis:

$$v_{meteoroid} = 7.5\sin(30°) = 3.75\ ms^{-1}$$

Calculate the momentum of the meteoroid in the y-axis is:

$$\rho_{meteoroid} = 10 \times 3.75 = 37.5\ kg\ ms^{-1}$$

Determine the momentum of the rocket in the y-axis is:

$$\rho_{rocket} = 0$$

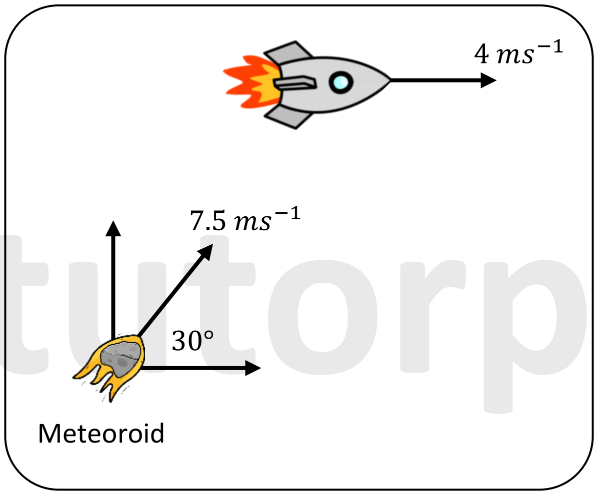
Calculate the total momentum in the y-axis is:

$$\rho_y = 37.5 + 0 = 37.5\ kg\ ms^{-1}$$

Collision vectors

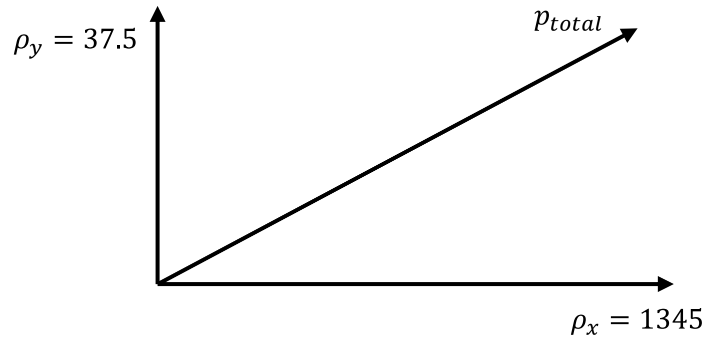
For example:

A spacecraft moves at $4\ ms^{-1}$ in space. A meteoroid, traveling at $7.5\ ms^{-1}$, collides with the spacecraft at a $30°$ angle and becomes embedded into it. The spacecraft has a mass of $320\ kg$ and the meteoroid has a mass of $10\ kg$. Calculate the spacecraft's velocity after the collision.



Collision vectors

Step 3: Resolve ρ_x and ρ_y into horizontal and vertical components, then find the resultant (p_{total}).



$$\rho_{total} = \sqrt{(\rho_x)^2 + (\rho_y)^2} = \sqrt{(1345)^2 + (37.2)^2}$$

$$\rho_{total} = 1345.5\ kg\ ms^{-1}$$

Step 4: Determine the new velocity after the collision by dividing the total momentum by the total mass of the rocket and meteorite.

$$v_{after} = \frac{\rho_{total}}{(m_{rocket} + m_{meteorite})} = \frac{1345.5\ kg\ ms^{-1}}{(320 + 10)}$$

$$v_{after} = 4.08\ ms^{-1}$$

Angle of momentum (i.e. direction of velocity) after collision:

$$\theta = \tan^{-1}\left(\frac{37.5}{1345}\right) = 1.60°$$

Therefore, the rocket with the embedded meteorite moves at $4.08\ ms^{-1}$ at an angle of $1.60°$ away from its original direction of motion.

Particle Momentum (Edexcel Only)

We can combine kinetic energy and momentum in order to produce a formula that gives kinetic energy in terms of momentum and mass.

Kinetic energy: $E_k = \frac{1}{2}mv^2 \;\ldots\ldots\ldots (1)$

Momentum: $\rho = mv$

Rearranging gives momentum for v gives us:

$$v = \frac{\rho}{m} \ldots\ldots\ldots (2)$$

Now substitute equation (2) into equation (1):

$$E_k = \frac{1}{2}m\left(\frac{p}{m}\right)^2$$

$$E_k = \frac{1}{2}\frac{p^2}{m}$$

$$\therefore E_k = \frac{p^2}{2m}$$

When dealing with kinetic energy of subatomic particles travelling at non-relativistic speed (speeds that are much slower than the speed of light) the above formula is very useful.

Please see 'Collisions and Explosions Worked Examples' pack for exam style questions.

Please see '6.2.2 Collisions and 2D Momentum Worked Examples' pack for exam style questions.

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