# AS Level Physics

Chapter 4 – Waves

4.4.1 Stationary Waves

Notes

# Stationary (or Standing) Waves

**Progressive waves travel outwards from a source and carry energy from one place to another through a material or a vacuum.**

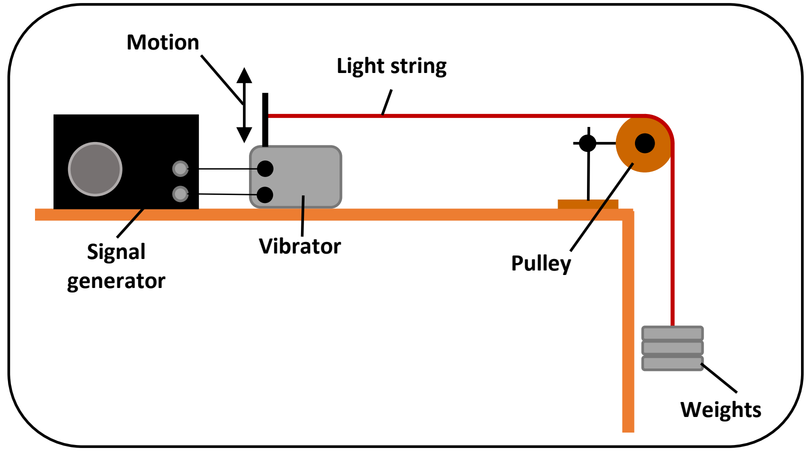
**A stationary (or standing) wave is the superposition of two identical progressive waves (with the same wavelength, speed, frequency, and roughly equal amplitudes) moving in opposite directions.**

In simple words, when a progressive wave is reflected at a boundary, stationary waves are produced.

A stationary wave, unlike progressive waves, does not transmit energy.

An object can reach its maximum amplitude at the resonant frequency. We will cover resonance in greater detail in a later pack.

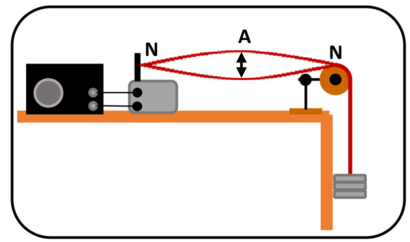
# Stationary Waves on a Stretched String




The above setup can be used to investigate the behaviour of vibrating strings. A string or wire is connected to the vibrator on one end. The vibrator is then connected to a signal generator. The other end of the string passes over a pulley and is supported by a weight, which keeps the tension in the spring constant.

The vibrator produces transverse waves along the string which are then reflected by the pulley. The reflected waves meet with the waves coming from the vibrator and superposition takes place. Therefore stationary waves are produced because the two sets of waves are similar and travel in opposing directions.

A stationary wave (or standing wave) pattern is created at particular frequencies (known as the Resonant Frequencies). A single loop with a large amplitude can be seen at this frequency, as shown opposite.




Also note, at both ends of the string you see Nodes $(N)$. The amplitude is zero at Nodes.

In the centre of the loop, an Antinode $(A)$ can be seen. These are points where the amplitude is at its maximum.

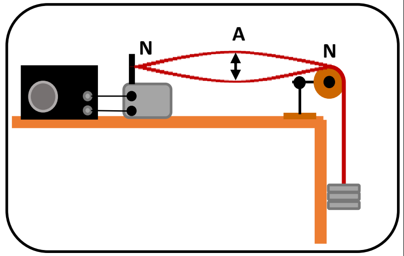
# Stationary Waves on a Stretched String

The lowest frequency at which a single loop forms a stationary wave is referred to as the:

**Fundamental Mode Of Vibration**

The frequency with which this occurs is referred to as:

**Fundamental Frequency ($f_0$)**

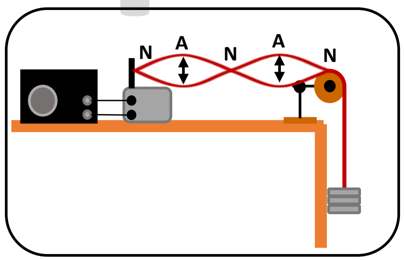



This stationary wave with a single loop at the fundamental mode of vibration can also be called the 1st Harmonic.

The stationary wave is generated because the vibrator is ready to send out a second wave in the time it takes for the first wave to reach the end and return. So, the second wave from the vibrator reinforces with the first wave. Each new input from the vibrator is in phase with the wave in the string, therefore increasing the amplitude, as the energy is continuously added to the string.

The single-loop stationary wave dissipates when the vibrator frequency is increased further.

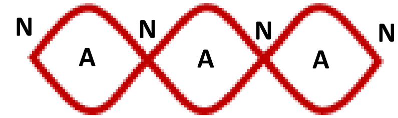
A new stationary wave is created with twice the fundamental frequency ($2f_0$). This wave has two loops with three nodes (N); one in the centre and one at each end of the string. There are also two antinodes (A) produced. This new stationary wave with 2 loops is known as the **2nd harmonic.**



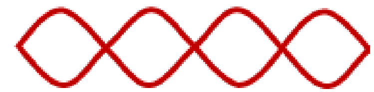



# Stationary Waves on a Stretched String

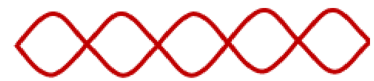
When the vibrator frequency is increased to $3f_0$, $4f_0$, $5f_0$, ...etc, more stationary waves with 3, 4, 5,..., vibrating loops are seen, example below:




$3f_0$



$4f_0$



$5f_0$

# Harmonics of Stretched Strings

So to summarise, when a stretched string is set into vibration, a progressive wave travels along the string to the fixed end where it is reflected. As these reflected waves (of equal frequency and amplitude) become superposed, a stationary wave forms on the string.

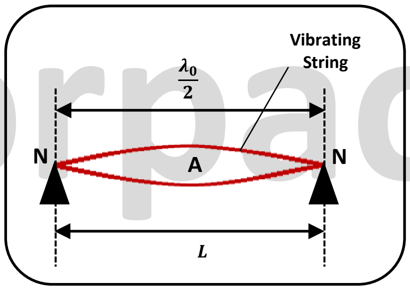
Different stationary wave patterns appear on the string as the frequency of the generator is increased from a very low value.

Furthermore, there is a node at either end of the string between the pulley and the vibrator in every situation.

When the stationary waves are formed, an exact number of half wavelengths $\left(\frac{1}{2}\lambda\right)$ fit onto the string at resonant frequencies.

## Fundamental Mode of Vibration (1st Harmonic)

At the lowest possible frequency, the fundamental pattern of vibration can be seen, with a node (N) at the fixed ends and an Antinode (A) in the middle. If L is the length of the string and $\lambda_0$ is the wavelength of the stationary wave, it can be seen that a string of length $L$ in the fundamental mode only produces half a wavelength $\left(\frac{1}{2}\lambda_0\right)$, so:




$$L = \frac{\lambda_0}{2}$$

$$\therefore \lambda_0 = 2L$$

We also know that $v = f\lambda$, thus rearranging this we can find frequency to be $f = \frac{v}{\lambda}$. Therefore, the fundamental frequency is calculated as follows:

$$f_0 = \frac{v}{\lambda_0}$$

Then substituting $\lambda_0 = 2L$ into the above formula, we can rewrite the fundamental frequency as:

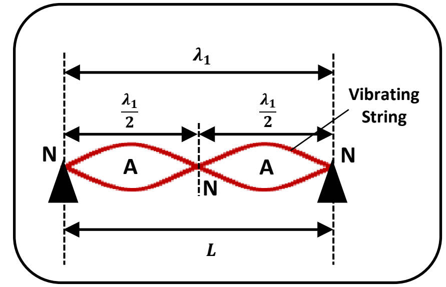
$$\boldsymbol{f_0 = \frac{v}{2L}}$$



Harmonics of Stretched Strings

- **2nd Harmonic**

The 2nd Harmonic will occur if the generator's frequency is increased. There is a node in the middle, and the string is divided into two loops.




Here each loop is half a wavelength long therefore $\lambda_1$ is L. Hence:

$$\lambda_1 = L$$

Therefore, the frequency of the 2nd harmonic vibrations is:

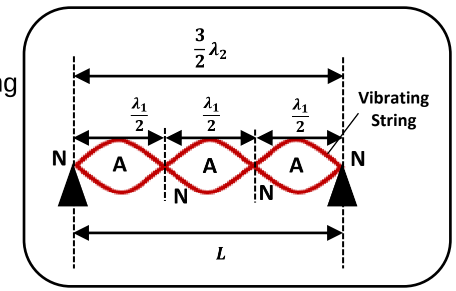
$$f_1 = \frac{v}{\lambda_1} = \frac{v}{L} = \boldsymbol{2f_0}$$

We know that $f_0 = \frac{v}{2L}$, therefore:

$$2 \times f_0 = 2 \times \frac{v}{2L} = \frac{v}{L} = f_1$$

- **3rd Harmonic**

The 3rd Harmonic is the next stationary wave pattern, with four nodes (N) and three antinodes (A).




Since each loop is half a wavelength long and there are three loops, you get:

$$\frac{3}{2}\lambda_2 = L$$

$$\therefore \lambda_2 = \frac{2}{3}L$$

As a result, the 3rd harmonic vibration's frequency is:

$$f_2 = \frac{v}{\lambda_2} = \frac{3v}{2L} = \boldsymbol{3f_0}$$

We know that $f_0 = \frac{v}{2L}$, therefore:

$$3f_0 = 3 \times \frac{v}{2L} = \frac{3v}{2L}$$

## Harmonics of Stretched Strings

You can see a pattern emerge as we go through the harmonics. Stationary wave patterns occur at frequencies such as:

Fundamental Mode (1st Harmonic) = $f_0$

2nd Harmonic = $2f_0$

3rd Harmonic = $3f_0$

and so on where $f_0$ is the frequency of the fundamental vibrations. This is true for every vibrating linear system with a node at both ends.

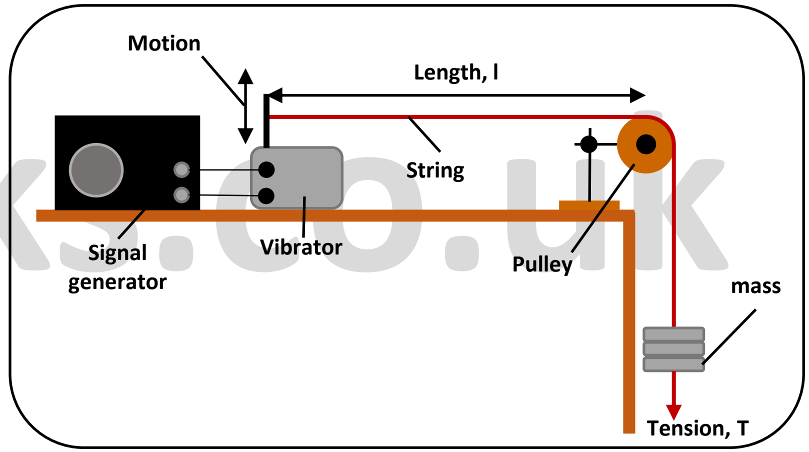


## Investigating Resonance (AQA only)

To investigate the impact of mass, length, and tension on the resonant frequencies of a string, follow these steps:

**Step1) Measure string mass and length:**

- Use a mass balance to measure the mass $(M)$ of different types of strings.
- Use a ruler to measure the length $(L)$ of each string.
- Calculate the mass per unit length $(\mu)$ of each string using the formula $\mu = \frac{M}{L}$, where $\mu$ is in $kg\ m^{-1}$, $M$ is in $kg$ and $L$ is in $m$.




**Step 2) Set up the apparatus:**

- Set up the experiment as shown above with one of the strings.
- Record the calculated $\mu$.
- Measure and record the length $(l)$ between the vibrator and the pulley.
- Calculate the tension $(T)$ on the string using $T = mg$, where $m$ is the added mass in kg, and $g$ is the acceleration due to gravity (approx. $9.81\ ms^{-2}$)

## Investigating Resonance (AQA only)

**Step 3) Find the first harmonic:**

- Turn on the signal generator and adjust the frequency to make the vibration transducer vibrate.
- Create the first harmonic, characterised by a stationary wave with a node at each end and a single antinode.
- Note the frequency $(f)$ of the first harmonic as indicated by the signal generator.

**Step 4) Investigate the effects:**

- To examine the effect of each factor (mass, length, and tension) on the resonant frequency, keep all other conditions constant while varying one factor at a time:
  - Change the length $(l)$: Move the vibrator closer to or farther from the pulley to alter the vibrating length of the string.
  - Change the tension $(T)$: Add or remove masses to adjust the tension on the string.
  - Change the string sample: Use strings of different masses to vary $\mu$



## Investigating Resonance (AQA only)

**Factors affecting resonant frequency**

The resonant frequency is impacted by the length, mass per unit length, and tension in the following manner:

- **Length Impact:** The longer the string, the lower its resonant frequency. This is due to the half-wavelength being longer, meaning if wavelength (λ) increases, frequency (f) decreases assuming speed (c) is constant.
- **Mass per Unit Length Impact:** A heavier string (greater mass per unit length) results in a lower resonant frequency, as waves move slower along the string. With a constant length, a lower wave speed (c) results in a lower frequency (f).
- **Tension Impact:** Lower tension on the string leads to a lower resonant frequency. This is because waves travel more slowly on a loose string.

# Investigating Resonance (AQA only)

**Calculate resonant frequency**

The frequency of the first harmonic of a string can be calculated using:

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

Where:

$f$ = frequency in $Hz$

$l$ = length of vibrating string in $m$

$T$ = tension on string in $N$

$\mu$ = mass per unit length of string in $kg\ m^{-1}$

The experiment confirms the formula: longer strings or higher mass decrease frequency, while greater tension increases it.

This formula applies only to the first harmonic, not to other frequencies.



# Air Columns (Pipes)

When the air at one end of a tube is caused to vibrate, a progressive, longitudinal wave travels down the tube and is reflected at the other end.

The diagram below shows a pipe with fine powder inside it. There is also a small loudspeaker linked to a signal generator near the open end of a pipe. When the signal frequency is adjusted, the pipe resonates with sound at certain frequencies. At each resonance stationary sound waves are formed in the pipe.

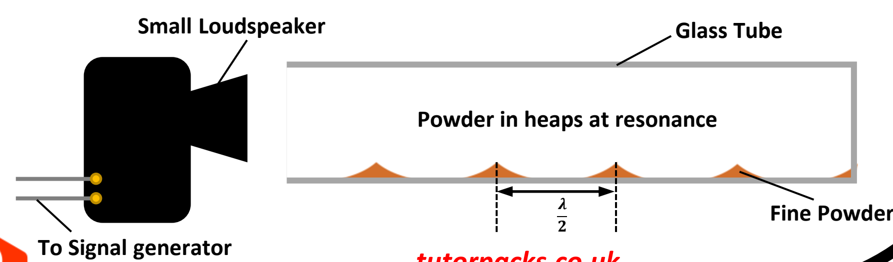
The sound waves are reflected at the closed end of the tube. The reflected waves go in opposite directions down the pipe, and because they have the same speed, frequency, and amplitude, they superpose and produce a stationary wave pattern at certain frequencies.

The fine powder in the tube creates evenly spaced piles at resonant frequencies. Since, air molecules move longitudinally down the tube axis, the vibrations amplitude changes from a maximum at the antinodes (A) to zero at the nodes (N).

The large amplitude vibrations at the antinodes positions shift the fine power, causing it to concentrate towards the node positions, where the amplitude of the molecules' vibrations is zero.

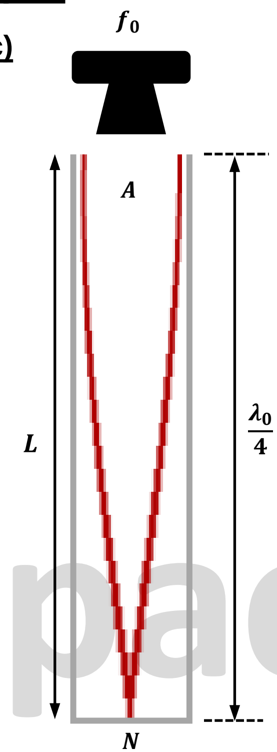
Remember:

- The amplitude of air molecule vibration is always maximum at the tube's open end (i.e. it is an Antinode).
- At the tube's closed end, the amplitude of air molecules' vibration is always zero (i.e. it is a Node).
- **Distance between adjacent nodes (or antinodes) =** $\frac{1}{2}\lambda$



# Harmonics of Closed Pipes

## Fundamental Mode of Vibration (1st Harmonic)




The lowest frequency at which the pipe resonates with sound (the fundamental frequency), there is a node (N) at the closed end and an antinode (A) at the open end.

As shown opposite, L is the pipe length and only a quarter of a wavelength $\left(\frac{1}{4}\lambda\right)$ is produced therefore:

$$L = \frac{\lambda_0}{4}$$

And the fundamental wavelength is,

$$\lambda_0 = 4L$$

Again using $f = \frac{v}{\lambda}$ we get:

So the fundamental frequency,

$$f_0 = \frac{v}{4L}$$

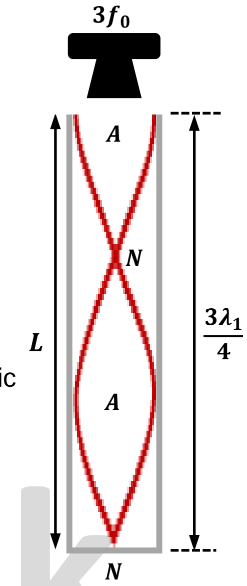
where $v$ is the speed of sound in the pipe.

The harmonics occurs at **odd number frequencies** such as $3f_0$, $5f_0$, $7f_0$, and so on. There is a node (N) at the closed end and an antinode (A) at the open end in each example, with one or more evenly spaced nodes (N) or antinodes (A) in between.



# Harmonics of Closed Pipes

## 3rd Harmonic




The 3rd Harmonic, you can see ¾ of a wavelength $\left(\frac{3\lambda_1}{4}\right)$ therefore:

$$L = \frac{3\lambda_1}{4}$$

**And so the 3rd harmonic wavelength,**

$$\boldsymbol{\lambda_1 = \frac{4L}{3}},$$

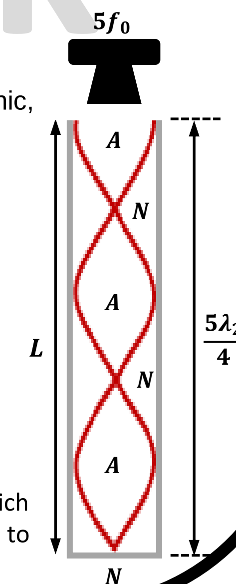
where $L$ is the pipe length.

The note produced has a frequency three times higher than the fundamental frequency because the 3rd harmonic wavelength is one third of the wavelength of the 1st harmonic. Thus a closed pipe can only produce odd harmonics and using $f = \frac{v}{\lambda}$ we get:

The 3rd Harmonic frequency, $\boldsymbol{f_1 = \frac{3v}{4L} = 3f_0}$

Where $v$ is the speed of sound in the pipe.

## 5th Harmonic




You can see 1 and ¼ of a wavelength at the 5th Harmonic, producing $\frac{5\lambda_2}{4}$ as a fraction therefore:

$$L = \frac{5\lambda_2}{4}$$

And so 5th harmonic wavelength, $\lambda_2 = \frac{4L}{5}$,

where L is the pipe length.

And using $f = \frac{v}{\lambda}$ we get:

The 5th Harmonic frequency, $\boldsymbol{f_2 = \frac{5v}{4L} = 5f_0}$

Where $v$ is the speed of sound in the pipe.

Additional resonances can be found at $7f_0$, $9f_0$, etc., which correspond to an odd number of quarter wavelengths equal to the pipe length.

# Harmonics of Open Pipes

An open pipe is one that is open to the atmosphere on both ends.

In an open pipe sound waves travelling along the pipe partially reflect at the open end because the speed changes at the exit. At either end, an antinode (A) is produced.

Consider, a low-pressure region travelling along the tube towards the open end. Since the air outside is at atmospheric pressure, when the low pressure region reaches the tube's end, air from the atmosphere rushes in, creating a compression wave that travels back down the tube. Vice verse is true when a high pressure region hits the end of the tube.

## Fundamental Mode of Vibration (1st Harmonic)

The lowest frequency at which the pipe resonates with sound (the fundamental frequency)**,** there is an antinode (A) at either end of the pipe with a node in the middle.

As you can see opposite, L is the length of the pipe and only half of a wavelength $\left(\frac{1}{2}\lambda\right)$ is produced therefore:

$$L = \frac{\lambda_0}{2}$$

So the fundamental wavelength is:

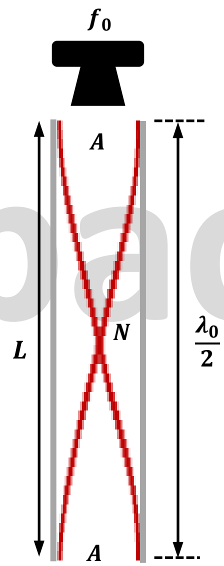
$$\lambda_0 = 2L$$

Using $f = \frac{v}{\lambda}$ we get:

The fundamental frequency,

$$f_0 = \frac{v}{2L}$$

Where $v$ is the speed of sound in the pipe.




The harmonics occurs at frequencies such as $2f_0$, $3f_0$, $4f_0$, and so on. Each harmonic, has an antinode (A) at either end, with one or more equally spaced nodes (N) or antinodes (A) in between.

# Harmonics of Open Pipes

## 2nd Harmonic

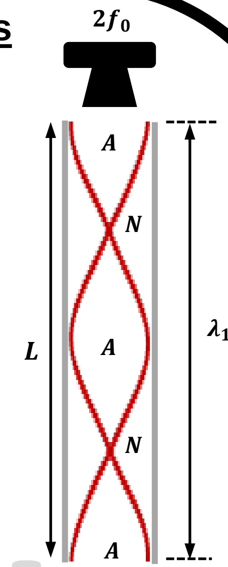
At the 2nd Harmonic, you see a whole wavelength ($\lambda_1$) therefore:

$$L = \lambda_1$$

**So 2nd harmonic wavelength, $\boldsymbol{\lambda_1 = L}$,**
Where L is the pipe length

Using $f = \frac{v}{\lambda}$ we get:

The 2nd Harmonic frequency, $\boldsymbol{f_1 = \frac{v}{L} = 2f_0}$
Where $v$ is the speed of sound in the pipe.




## 3rd Harmonic

At the 3rd Harmonic, you can see one and a half wavelength producing $\boldsymbol{\frac{3\lambda_2}{2}}$ as a fraction therefore:

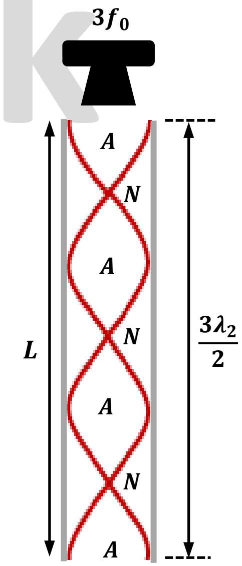
$$L = \frac{3\lambda_2}{2}$$

And so 5th harmonic wavelength, $\lambda_2 = \frac{2L}{3}$,
Where L is the pipe length

Using $f = \frac{v}{\lambda}$ we get:

The 3rd Harmonic frequency, $\boldsymbol{f_2 = \frac{3v}{2L} = 3f_0}$
Where $v$ is the speed of sound in the pipe.

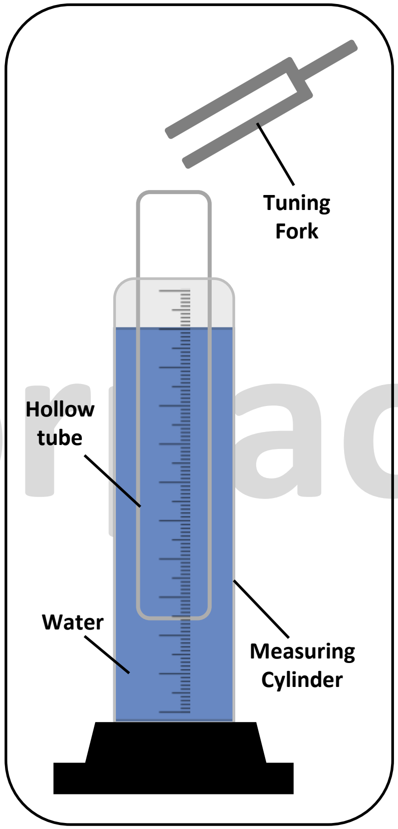
Further resonances occur at $4f_0$, $5f_0$, etc.,



# Finding the Speed of Sound in a Air Pipe

A closed-end pipe can be made by putting a hollow tube into a measuring cylinder of water. Holding a sounding tuning fork over the open end of the tube will cause it to vibrate.

The hollow tube has a natural frequency of vibration, and if the tuning fork frequency matches it, the tube enters into resonant vibration, and the tuning fork sound becomes considerably louder.



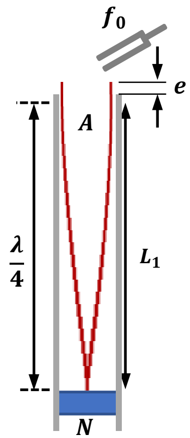

When choosing a tuning fork, make a note of the frequency of sound it generates (it will be stamped on the side).

Tap the tuning fork gently and hold it directly above the hollow tube. The sound waves produced by the tuning fork travel down the tube, where they are reflected (forming a node) at the air/water surface.

Move the tube up and down until the sound from the fork resonates at a shorter distance between the top of the tube and the water level, i.e. adjust the tube length until a LOUD sound is heard.



# Finding the Speed of Sound in a Air Pipe




The lowest frequency at which the pipe resonates with sound (the fundamental frequency or the first resonance), there is a node (N) at the closed end and an antinode (A) at the open end.

As you can see opposite, L is the pipe length and only a quarter of a wavelength $\left(\frac{1}{4}\lambda\right)$ is produced therefore:

$$L_1 = \frac{\lambda}{4} \ldots \ldots \ldots (1)$$

Therefore fundamental wavelength, $\lambda = 4L_1$

By increasing the length of the air column further, a second resonance position with three times the fundamental frequency is obtained.

You can see three-quarters of a wavelength $\left(\frac{3\lambda}{4}\right)$ therefore:

$$L_2 = \frac{3\lambda}{4} \ldots \ldots \ldots (2)$$

**So 2nd harmonic wavelength is, $\boldsymbol{\lambda = \frac{4L_2}{3}}$**

Where L is the pipe length

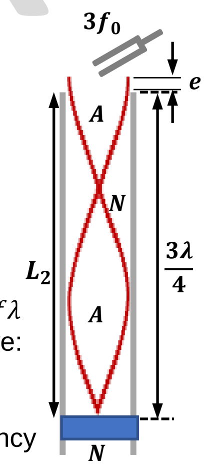
Now $(2) - (1) \longrightarrow L_2 - L_1 = \frac{\lambda}{2}$

$$\therefore \ \lambda = 2(L_2 - L_1)$$

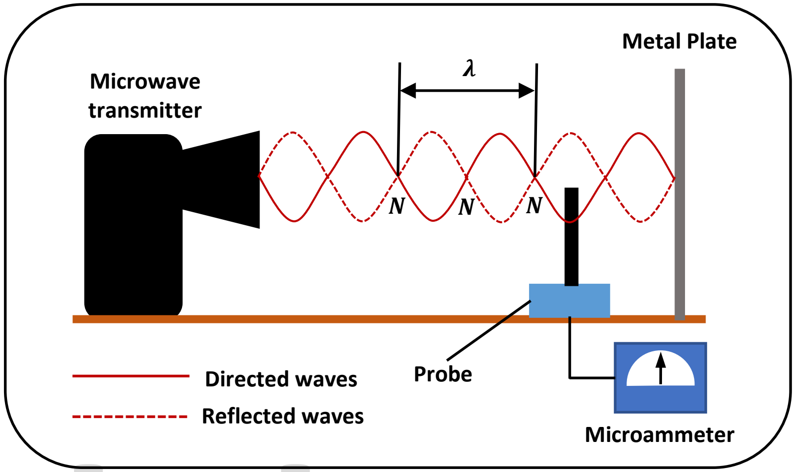
That is our wavelength of the sound in the pipe and using $v = f\lambda$ we can calculate the speed of sound by rearranging for $\lambda$ to give:

$$v = 2f(L_2 - L_1)$$

Where, $v$, is the speed of sound in the pipe and $f$ is the frequency of the tuning fork.



# Demonstration of Stationary Waves with Microwaves






Microwaves can be directed towards a metal plate using a microwave transmitter attached to a signal generator, as seen above. At the metal plate, the microwaves are reflected back to the emitter.

Here, low readings are detected at evenly spaced positions when a probe linked to a microammeter is moved along the line between the transmitter and the metal plate.

This is due to the fact that the reflected waves and the directed waves from the transmitter combine to form a stationary wave. When a microammeter reading is low, it means the microwave intensity is low, and a Node, $N$, is formed; when the reading is high, it means the microwave intensity is high, and an Antinode, $A$, is formed.

A full wavelength contains three nodes, so the distance that the probe moves through a number of nodes can be used to determine the microwaves wavelength $(\lambda)$. This can be done by measuring the distance travelled by the probe as it passes through three nodes indicated by minimum microammeter values.



# Demonstration of Stationary Waves with Microwaves

The signal generator provides the value for the frequency $(f)$, thus microwaves speed $(v)$ can be calculated using:

$$v = f\lambda$$

Please see '4.4.2 Stationary worked examples' pack for exam style questions.

For more revision notes, tutorials and worked examples please visit www.tutorpacks.co.uk.