A2 Level Physics

Chapter 9 - Thermodynamics

9.4.1 Ideal Gases

Notes

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Avogadro Constant and The Mole

We measure the amount of a substance in a unit called the 'mole'. It is abbreviated to mol. This is a simple way of counting atoms in a substance which in turn allows us to predict the masses of various substances that are involved in reactions.

1 mol is the mass of a substance that contains 6.022×10^{23} particles.

A substance's molarity is the number of moles contained in a given amount of the substance.

One mole contains 6.022×10^{23} atoms, no matter what element it is. This is a large number so we have given it a name and called it the Avogadro constant:

$$\boldsymbol{The\ Avogadro\ constant,\ N_A = 6.022 \times 10^{23}\ atoms\ per\ mole}$$

or

$$\boldsymbol{Avogadro\ constant, N_A = 6.022 \times 10^{23}\ mol^{-1}}$$

Avogadro constant is named after Amadeo Avogadro who put forward the hypothesis that equal volumes of gases at the same temperature and pressure contain an equal number of molecules. This is where the idea of how many molecules are there in a certain amount of gas got raised.

This number is used because if you could count out 6.022×10^{23} carbon atoms, the total mass of carbon is equal to 12g. By weighing out 12g, on the other hand, you can determine how many atoms you have.

So one mole of carbon-12 isotope has a mass of 12g and contains 6.02×10^{23} carbon-12 atoms.

One mole of helium-4 isotope contains 6.02×10^{23} helium-4 atoms and has a mass of 4g.

One mole of magnesium-24 contains 6.02×10^{23} magnesium-24 atoms and has a mass of 24g.

In fact, the mass of one mole of any element is equal to its relative atomic mass in grams. A mole of oxygen weighs 16 grams.

However, many gases are found not as single atoms. For example:

$$1\ mol\ of\ ozone\ (O_3) = 6.02 \times 10^{23} molecules = 3 \times 6.02 \times 10^{23} atoms$$
$$\therefore 1\ mol\ of\ ozone\ (O_3) = 18.06 \times 10^{23} atoms$$

Avogadro Constant and The Mole

The Avogadro constant, N_A, is defined as the number of atoms in exactly 12 grams of carbon isotope $^{12}_{6}C$.

A chemical compound's molar mass (M) is defined as the mass of a sample divided by the amount of substance in that sample measured in moles.
The unit of molar mass is $kg\ mol^{-1}\ or\ g\ mol^{-1}$.

For example, the molar mass of oxygen (O_2) is $0.032\ kg\ mol^{-1}$ or $32\ g\ mol^{-1}$. So 0.032kg or 32g of oxygen contains 6.02×10^{23} oxygen molecules.

To calculate the number of moles (n) of any substance divide the given mass (m) of the substance by its molar mass (M). So:

$$n = \frac{m}{M}$$

To calculate the number of molecules (N) in n moles of gas we use:

$$N = nN_A = \frac{m}{M} N_A$$

The relative atomic mass (M_r) of an element is the average mass of its atoms, compared to 1/12th the mass of a carbon-12 atom.
Relative atomic mass of carbon is 12.
Relative atomic mass of oxygen is 16.
Relative formula mass of carbon dioxide CO_2 is:
$12 + (16 \times 2) = 44$

The Ideal Gas Equations

An ideal gas obeys the three gas laws, which through experiments prove that a fixed mass of any gas obeys the relationships listed below:

- At constant temperature $pressure \propto \frac{1}{Volume}$ (also known as Boyle's Law).
- At constant pressure $Volume \propto Temperature$ (also known as Charles' Law).
- At constant volume $pressure \propto Temperature$ (also know as the Pressure Law).

Let's look at the relationships more closely.

Boyle's Law

The Boyle's law states:

At constant temperature, the pressure (p) of a fixed mass of gas is inversely proportional to its volume (V)

$$p \propto \frac{1}{V}$$

Or:

$$pV = constant$$

When a fixed mass of gas is compressed into a smaller volume (at higher pressure) or allowed to expand into a larger volume (by reducing pressure), this equation can be used to calculate pressure or volume changes, as long as the temperature remains constant throughout the change.

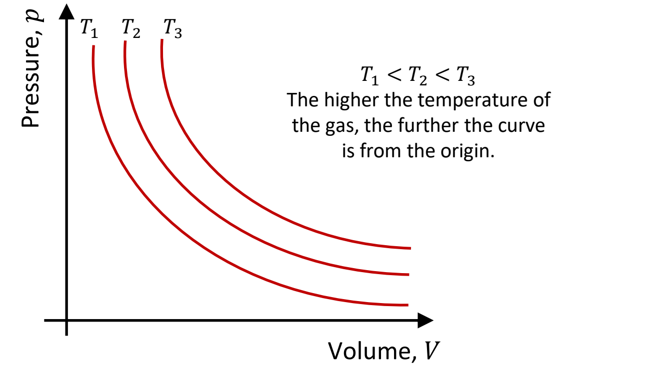
If (p_1) and (V_1) are the initial pressure and volume of a fixed mass of gas, and (p_2) and (V_2) are the values after expansion or compression at constant temperature. Then:

$$p_1V_1 = p_2V_2$$

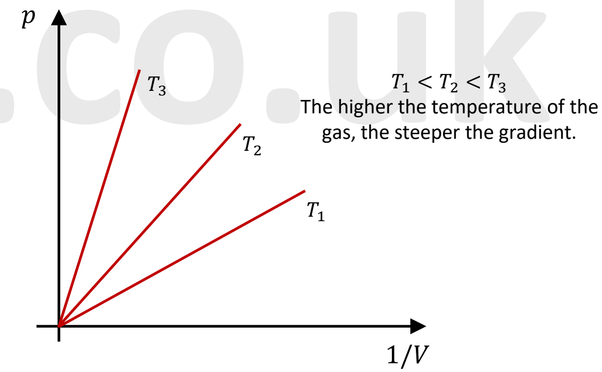
An ideal gas is a (theoretical) gas that obeys Boyle's law at all temperatures.

The Ideal Gas Equations

The following is a graphical representation of the relationship between pressure (p) and volume (V) of a fixed mass of gas at constant temperature:



When p is plotted against $\frac{1}{V}$, a straight line graph is obtained:



Each of the curves or lines is called ISOTHERMAL (since the plotted values are all for the same temperature). In other words:

$$ISOTHERMAL = constant\ temperature$$

The Ideal Gas Equations

Boyle's Law Experiment

Apparatus:

- Air column with measuring scale,
- Pressure gauge,
- Oil reservoir,
- Hand air pump

Method:

Set up the experiment indicated opposite to investigate the effect of pressure on volume (Boyle's law) of a fixed mass of gas at constant temperature.

- A fixed mass of air is trapped by oil in a sealed tube with fixed dimensions. A volume scale is positioned against the sealed tube.
- First, increase the pressure in the oil with a tyre pump.
- Record the pressure using the pressure gauge. The pressure gauge measures the pressure in pascals. As the pressure increases, more oil is pushed into the tube, the oil level rises, and the air compresses. Therefore the volume of air in the tube will decrease.
- Use the scale positioned against the tube to determine the volume of the air column.
- Then, while keeping the temperature constant, gradually increase the pressure by a set interval.
- Keep recording the values for the pressure (p) and the volume (V) of air as it changes. At any time, multiplying these values should get the same result.
- Repeat the experiment twice more and take a mean for each reading.

If you plot p against $\frac{1}{V}$ on a graph, you should get a straight line.

The Ideal Gas Equations

Boyle's Law Experiment

Precautions:

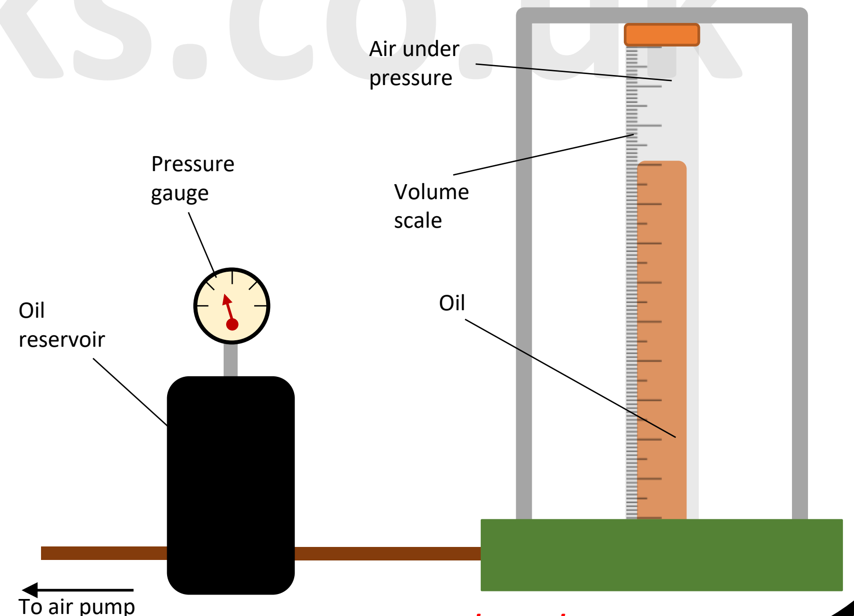
- Allow time for the gas to reach room temperature before making each measurement.
- Read the volume from the bottom of the meniscus.

Sources of error:

- Parallax error associated with measuring the pressure from the pressure gauge and volume from the volume scale.
- Variation in room temperature may affect results.

Improvements:

- Use a digital pressure gauge.
- Test Boyle's law at several different temperatures.



The Ideal Gas Equations

Charles' Law

Charles' law states that:

At constant pressure, the volume (V) of a fixed mass of gas is directly proportional to its absolute temperature (T).

$$V \propto T$$

If Charles' law is obeyed, the volume divided by the temperature is a constant:

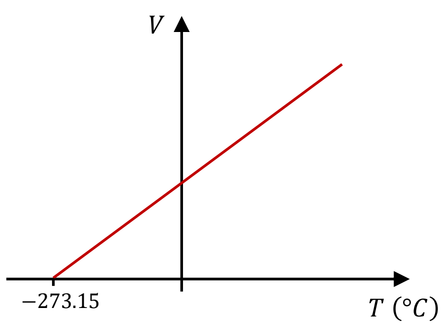
$$\frac{V}{T} = constant$$

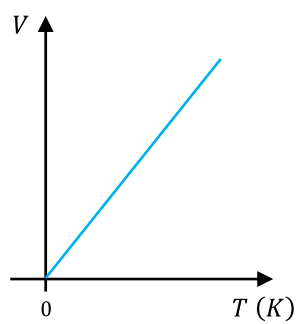
From which:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

An ideal gas also obeys Charles' law. The volume of an ideal gas is zero at the lowest theoretically possible temperature (absolute zero, $0K$).

The following graphs represent the relationship between the volume (V) and temperature (T) of a fixed mass of gas at constant pressure on the degree Celsius scale and on the Kelvin scale:





The Ideal Gas Equations

Charles' Law

In practice a real (non-ideal) gas would condense before reaching $0K$.

When a gas is heated, the particles gain kinetic energy and move faster. This implies the particles move further apart at constant pressure, increasing the volume of the gas.

The Ideal Gas Equations

Charles' Law Experiment

Apparatus:

- Capillary tube
- Sulfuric acid
- Beaker
- Ruler
- Thermometer
- Kettle

Method:

- Set up the experiment indicated opposite to investigate the effect of temperature on volume (Charles' law) of a fixed mass of gas at constant pressure.
- First get a capillary tube containing a drop of concentrated sulfuric acid positioned halfway up the tube. The capillary tube's bottom should be sealed, trapping a small column of air between the tube's bottom and the acid drop.
- In a beaker of hot water, place the capillary tube. Then, place a ruler behind the capillary tube to measure the length of the air column trapped between the tube's bottom and the drop of sulfuric acid (see opposite).
- As the water cools, in regular intervals, record the temperature of the water and the length of the air column. The air pressure above the droplet is assumed to be constant.
- Repeat the experiment two more times with fresh near-boiling water, allowing the capillary tube to adjust to the new temperature between each repeat, for a total of three temperature readings. For each temperature, average the length of the air column.
- You should see that when the water temperature drops, the length of the trapped air column decreases.
- Plot your results on a graph of length against temperature and draw a line of best fit – you should get a straight line. This shows that the length of the air column is proportional to the temperature.
- The volume of the air column is equal to the volume of a cylinder, $V = \pi r^2 l$, where r is the radius of the column and l is its length. In this experiment, r^2 remains constant, indicating that the length is proportional to the volume of the air column, and hence the volume is proportional to the temperature. This agrees with Charles' law.

The Ideal Gas Equations

Charles' Law Experiment

Safety:

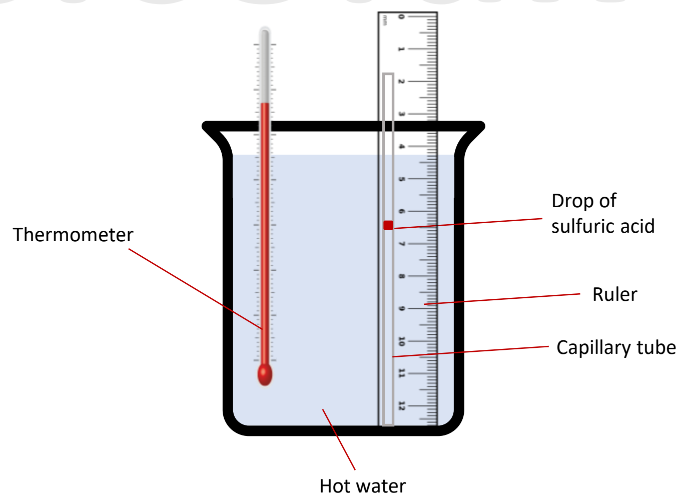
- Since concentrated sulfuric acid can cause eye damage, safety goggles must be worn.
- The usage of boiling water may result in burns, therefore take care not to spill it.

Sources of error:

- Parallax error associated with measuring the temperature from the thermometer and sulfuric acid from the volume scale.

Improvements:

- Use a digital thermometer.
- To avoid the sulfuric acid thread from splitting, the tube must be completely clean with no traces of other chemicals.



The Ideal Gas Equations

Pressure Law

Pressure law states:

At constant volume, the pressure (p) of a fixed mass of gas is directly proportional to its absolute temperature (T).

$$p \propto T$$

If pressure law is obeyed, the pressure divided by the temperature is a constant:

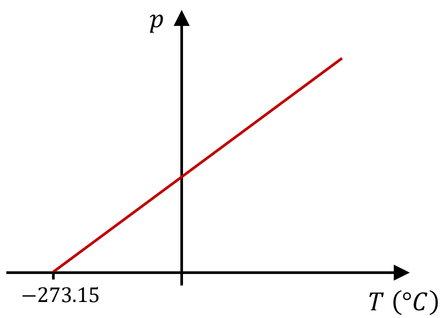
$$\frac{p}{T} = constant$$

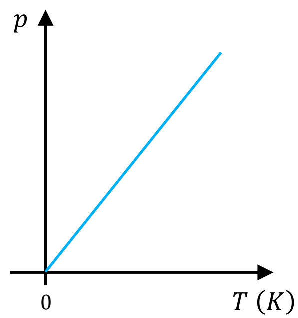
From which:

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

An ideal gas obeys the pressure law. The pressure is zero at the lowest theoretically possible temperature (absolute zero, 0K).

The following graphs represent the relationship between the pressure (p) and temperature (T) of a fixed mass of gas at constant volume on the degree Celsius scale and on the Kelvin scale:





The Ideal Gas Equations

Pressure Law

In practice a real (non-ideal) gas would condense before reaching $0K$.

The particles in a gas gain kinetic energy as it is heated. This indicates that they are moving at a faster rate. If the volume does not change, the particles will collide more frequently and at higher speeds with each other and their container, raising the pressure inside the container.

The Ideal Gas Equations

Pressure Law Experiment

Apparatus:

- Pressure gauge
- Large beaker
- Flask
- Thermometer
- Cork
- Heater

Method:

Set up the experiment presented opposite to investigate the effect of temperature on pressure (pressure law) of a fixed mass of gas at constant volume.

Heat a specified volume of gas in a water bath and use a pressure gauge to measure the pressure.

Start with water at 0°C (ice can be used) and gradually heat it up, checking the pressure of the gas in regular intervals - the gas temperature should be the same as the water.

Continue to heat the water until it reaches 100°C.

Plot the pressure against temperature on a graph and extrapolate to the point it crosses the x-axis. This is the absolute zero ($0K$ or -273°C).

This is a pressure and temperature experiment that can also be used to determine the absolute zero.

Safety:

- The usage of boiling water may result in burns, therefore take care not to spill it.

The Ideal Gas Equations

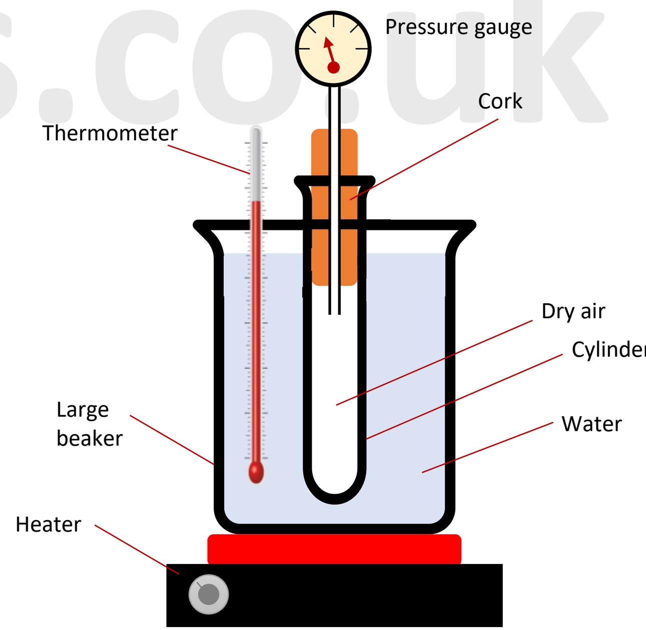
Pressure Law Experiment

Sources of error:

- Parallax error associated with measuring the temperature from the thermometer.

Improvements:

- Use a digital pressure gauge and thermometer.
- Make sure the flask is completely submerged in water and held above the bottom.
- Do not allow the thermometer to come into contact with the flask.
- Make sure there are no leaks in the flask or valve.



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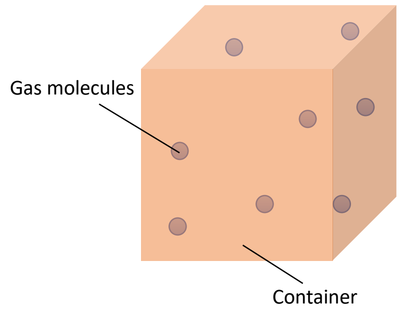
Model of Kinetic Theory of Gases

We're talking about a macroscopic description of a gas when we talk about its large-scale properties like mass, volume, temperature, and pressure.

The microscopic description consists of a large number of randomly moving molecules. The molecules themselves can be modelled as little unbreakable spheres that collide with one other and the container's walls.



The kinetic theory of gases relates the microscopic and macroscopic properties of a gas and is governed by the following basic assumption:

- A gas consists of a large number of identical molecules in a state of rapid, random motion.
- The collisions between molecules and the walls of the containing vessel are perfectly elastic.
- The forces between molecules are negligible (expect during collisions between molecules).
- The volume of the molecules is negligible compared with the total volume occupied by the gas.

An ideal gas also follows the assumptions above.

Pressure Exerted by a Gas

Pressure – exerted on a surface is defined as the perpendicular force per unit area of the surface.

$$Pressure(p) = \frac{Force(F)}{Area(A)}$$

Where:

- p = pressure measured in Pascals (Pa) or Nm^{-2},
- F = force measured in Newtons, N
- A = perpendicular area measured in m^2

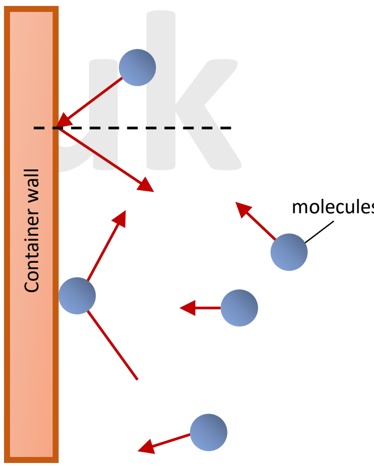
Gas pressure explained in terms of molecules:

Consider a gas that is contained within a container.

The gas molecules travelling at different speeds and in different directions collide with each other and with the container's walls continuously.

It is assumed that these collisions are perfectly elastic.

When a gas molecule collides with a wall, its momentum changes, exerting a small force on the wall.



Since there are so many molecule collisions every second, the overall outcome is that the gas exerts a noticeable pressure on the container's walls in macroscopic terms therefore:

$$Pressure = \frac{sum\ of\ forces\ due\ to\ all\ molecules}{Area\ of\ wall}$$

The ideal gas equation

An ideal gas is one that obeys the three gas laws and is subjected to the kinetic theory of gases' assumptions. As a result, for an ideal gas, the following is true:

- At constant temperature $pV = constant$ (Boyle's Law)
- At constant pressure $\frac{V}{T} = constant$ (Charles' Law)
- At constant volume $\frac{p}{T} = constant$ (Pressure Law)

When we combine the three laws, we get:

$$\frac{pV}{T} = constant$$

The size of the constant is determined by the mass of gas under consideration.

By putting in values for 1 mole of an ideal gas at room temperature and atmospheric pressure gives the constant a value of $8.31\ JK^{-1}mol^{-1}$. This is the molar gas constant, R.

Therefore:

$$\frac{pV}{T} = R$$

However, if there is more or less gas present, the value of pV/T increases or decreases – the more gas you have, the more space it takes up. Since the amount of gas is measured in moles, n, the constant in the above equation becomes nR, where n is the number of moles of gas present. Therefore:

$$\frac{pV}{T} = nR$$

The ideal gas equation

We can rearrange the ideal gas equation for n moles to be:

$$pV = nRT$$

Where:

p = pressure measured in Pascals, Pa

V = volume measured in m^3

n = number of moles of gas

R = molar gas constant $(= 8.31 JK^{-1}mol^{-1})$

T = absolute temperature measured in kelvins, K.

At all temperatures and pressures, an ideal gas would obey this equation. At room temperature and pressure, real gases like hydrogen, helium, and oxygen also obey this equation. However, when the temperature is drastically reduced or the pressure is extremely high, they no longer behave in this manner.

The ideal gas equation

The Boltzmann Constant, *k*

The Boltzmann constant, k, is equivalent to $\frac{R}{N_A}$:

$$k = \frac{R}{N_A}$$

Where:

k = the Boltzmann constant

R = molar gas constant $(= 8.31\ JK^{-1}mol^{-1})$

N_A = Avogadro's constant $(= 6.022 \times 10^{23} mol^{-1})$

The value of the Boltzmann constant is:

$$k = 1.38 \times 10^{-23}\ JK^{-1}$$

The Boltzmann constant can be thought of as the gas constant for one molecule of gas, whereas the molar gas constant (R) is the gas constant for one mole of gas.

The ideal gas equation

The Boltzmann Constant, *k*

We can express the ideal gas equation in terms of the Boltzmann constant:

Since: $k = \frac{R}{N_A}$ then $R = kN_A$

Substituting for R in $pV = nRT$ gives:

$$pV = n(kN_A)T$$

But we also know that $nN_A = N$ from page 2

Therefore:

$$pV = NkT$$

Where:

p = pressure measured in Pascals, Pa

V = volume measured in m^3

N = number of molecules in the gas

k = Boltzmann constant $(= 1.38 \times 10^{-23}\ JK^{-1})$

T = absolute temperature measured in kelvins, K.

This version of the ideal gas equation is useful for problems involving molecules rather than moles of gas.

The Combined Gas Equation

For (n) moles of a gas of volume (V_1) at a pressure (p_1) and temperature (T_1):

$$\frac{p_1 V_1}{T_1} = nR$$

For the same amount of gas (n) moles whose volume has changed to (V_2) at a new pressure (p_2) and temperature (T_2):

$$\frac{p_2 V_2}{T_2} = nR$$

We can deduce the following from the two equations:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

This is called the combined gas equation.

It's especially useful for solving problems where volume, pressure, and temperature all change at the same time.

It doesn't matter what units p and V are in as long as they're the same on both sides of the equation; however, T must be in Kelvins (K).

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Kinetic Theory and the Pressure of an Ideal Gas

Deriving the pressure of an ideal gas

Consider a gas molecule with mass, m in a cubic box of side L travelling at speed u_x parallel to the base of the box.

When the molecule collides with the right-hand wall, it bounces back with velocity $-u_x$, resulting in a change of momentum of:

$$change\ in\ momentum = mu_x - (-mu_x) = 2mu_x$$

The molecule travels a distance of 2L before colliding with the same wall again, so the time elapsed is:

$$t = \frac{2L}{u_x}$$

Using this we can calculate the rate of change of momentum:

$$\frac{\Delta p}{\Delta t} = \frac{2mu_x}{\frac{2L}{u_x}} = \frac{mu_x^2}{L}$$

Newton's 2nd Law states that force equals to the rate of change of momentum.

Therefore: Force applied by the molecule on this wall, $F = \frac{mu_x^2}{L}$

The area of the wall is L^2, so

$$Pressure = \frac{F}{A} = \frac{mu_x^2}{L^3}$$

Since a gas molecule can move in three dimensions, you must consider all three directions (x, y and z) for a more general equation. You can calculate the molecules speed, c, using Pythagoras' theorem: $c^2 = u_x^2 + u_y^2 + u_z^2$ where u_x, u_y and u_z are components of the molecule's velocity in the x, y and z directions, respectively.

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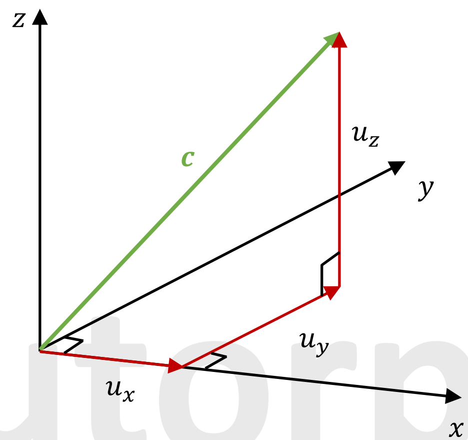
Kinetic Theory and the Pressure of an Ideal Gas

Deriving the pressure of an ideal gas

If all N molecules are treated in the same way, the overall mean square speed, $\overline{c^2}$, is:

$$\overline{c^2} = \overline{u_x^2} + \overline{u_y^2} + \overline{u_z^2}$$



Because the molecules move at random, $\overline{u_x^2} = \overline{u_y^2} = \overline{u_z^2}$. So $\overline{c^2} = 3\overline{u_x^2}$.

Therefore:

$$\overline{u_x^2} = \frac{1}{3}\overline{c^2}$$

You can substitute this into the equation for pressure below:

$$Pressure = \frac{F}{A} = \frac{mu_x^2}{L^3}$$

As a result, we get the below equation for all molecules:

$$p = \frac{1}{3}\frac{N m \overline{c^2}}{L^3}$$

Kinetic Theory and the Pressure of an Ideal Gas

Deriving the pressure of an ideal gas

But $L^3 = V$, the volume of the box, so we get:

$$p = \frac{1}{3}\frac{N m \overline{c^2}}{V}$$

Rearranging the equation we get:

$$\boldsymbol{pV = \frac{1}{3} N m \overline{c^2}}$$

Where:

p = pressure in Pa

V = volume in m^3

N = number of molecules of gas

m = mass of a gas molecule in kg

$\overline{c^2}$ = mean square speed of gas molecules in $m^2 s^{-2}$

Root mean square speed

The mean square speed $\left(\overline{c^2}\right)$ is measured in m^2s^{-2}. $\overline{c^2}$ is the mean of the squares of the speeds of molecules – the square root of it gives you the typical speed.

This is known as the root mean square speed or usually, the r.m.s. speed. The unit is the same unit as any speed, ms^{-1}.

$$r.m.s.\,speed = \sqrt{mean\ square\ speed} = \sqrt{\overline{c^2}} = c_{rms}$$

Therefore we get:

$$pV = \frac{1}{3}Nm(c_{rms})^2$$

Kinetic Theory and the Pressure of an Ideal Gas

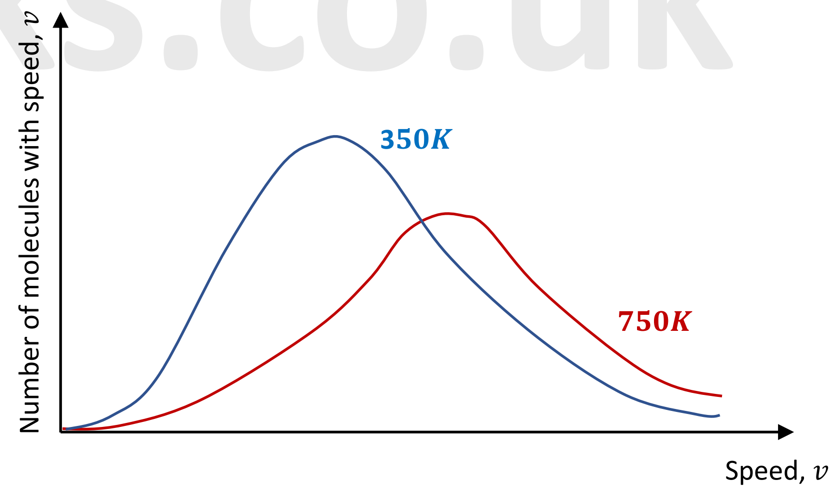
Molecular Speeds

As shown below, the molecules in an ideal gas have a continuous range of speeds.

The graph shows that a few molecules have extremely low or extremely high speeds. When a gas molecule collides with another, its speed changes, but the distribution remains the same as long as the temperature does not change.

When a gas's temperature increases, its molecules move faster on average. The mean speed of the molecules increases. As more molecules move at higher speeds, the distribution curve flattens and broadens.

The mean speed $(\bar{c})$ is the average value of the speeds of all the molecules.



Molecular Kinetic Energy and Temperature

Kinetic energy is about the energy in moving particles. It's not practical to calculate the individual kinetic energies of every single particle in a body of gas, but the average and total kinetic energy can be easily calculated.

You can assume that all of the internal energy in an ideal gas is in the form of kinetic energy. This is because the particles in an ideal gas do not interact (unless they collide), so there is no potential energy in an ideal gas. This means you can calculate the average and total kinetic energy using the product of pV.

For (n) moles of an ideal gas of volume (V) at a pressure (p) and absolute temperature (T), the equation of state is:

$$pV = nRT \ldots\ldots\ldots\ldots\ldots (1)$$

Using the kinetic theory of gases, if the gas is made up of (N) molecules each of mass (m) and moving with a mean speed $(\bar{c})$:

$$pV = \frac{1}{3}Nm\bar{c}^2 \ldots\ldots\ldots\ldots\ldots (2)$$

Combining (1) and (2) gives:

$$\frac{1}{3}Nm\bar{c}^2 = nRT$$

Which may be expressed as:

$$\frac{2}{3}N\left(\frac{1}{2}m\bar{c}^2\right) = nRT$$

Therefore:

$$\frac{1}{2}m\bar{c}^2 = \frac{3n}{2N}RT = \frac{3}{2}\frac{R}{N/n}T$$

But $\frac{N}{n} = N_A = Avogadro\ Number$

Molecular Kinetic Energy and Temperature

$$\frac{1}{2}m\bar{c}^2 = \frac{3}{2}\frac{R}{N_A}T \ldots\ldots\ldots\ldots\ldots (3)$$

R is the gas constant per mole of gas molecules.

$$\frac{R}{N_A} = k = Boltzmann\ constant.$$

$$E = \frac{3}{2}kT$$

Where:

E = mean translational kinetic energy, J

k = Boltzmann constant, $1.38 \times 10^{-23}\ J\ K^{-1}$

T = absolute temperature, K

Therefore:

The average translational kinetic energy (E) of an ideal gas molecule is proportional to the gas's absolute temperature (T).

From equation (3):

$$\bar{c}^2 \propto T$$

i.e. a gas molecule's mean-square speed is proportional to its absolute temperature.

Since an ideal gas's internal energy is considered to be purely kinetic:

The sum of the mean kinetic energies of all the molecules in a gas is the internal energy of the gas.

Please see '9.4.2 Ideal Gases worked examples' pack for exam style questions.

For more revision notes, tutorials and worked examples please visit www.tutorpacks.co.uk.