A2 Level Physics

Chapter 10 Nuclear Radiation

10.2.2 Nuclear Fission and Fusion

Worked Examples

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Nuclear Fission/Fusion and Binding Energy

Exam Style Question 1

(a) Explain what is meant by the binding energy of a nucleus.

(b) The fusion of protons occurs in a star when the temperature within the core is greater than about $10^7\ K$. It takes the fusion of 4 protons to form a helium-4 $\left({}_2^4He\right)$ nucleus. In this process, known as the proton–proton cycle, energy is released.

The net energy released in producing a single helium-4 nucleus is $4.53 \times 10^{-12}\ J$. Calculate the binding energy per nucleon of the helium-4 nucleus.

(c) The fusion of helium nuclei to make heavier elements occurs in red giants at temperatures above $10^8\ K$.

Explain why fusion of helium requires higher temperatures than the fusion of hydrogen (protons).

(d) Estimate the mean speed of helium nuclei at a temperature of $10^8\ K$.

mass of helium nucleus = $6.6 \times 10^{-27}\ kg$

Nuclear Fission/Fusion and Binding Energy

Exam Style Question 1

(a)Explain what is meant by the binding energy of a nucleus.

The minimum energy needed to separate the nucleus into its constituent protons and neutrons

(b) Calculate the binding energy per nucleon of the helium-4 nucleus.

$$BE\ per\ nucleon = \frac{4.53 \times 10^{-12}\ J}{4}$$
$$BE\ per\ nucleon = 1.13 \times 10^{-12}\ J\ (2\ d.p.)$$

(c) Explain why fusion of helium requires higher temperatures than the fusion of hydrogen (protons).

The helium nucleus has a greater charge and therefore experiences a greater repulsive force. So the higher temperature ensures that the helium nuclei comes close together for strong force to initiate fusion.

(d) Estimate the mean speed of helium nuclei at a temperature of $10^8\ K$.

mass of helium nucleus = $6.6 \times 10^{-27}\ kg$

Use $\frac{1}{2}mv^2 = \frac{3}{2}kT$ and rearrange to find v

$$v^2 = \frac{\frac{3}{2}kT}{\frac{1}{2}m}$$
$$v^2 = \frac{\left(\frac{3}{2}\right)(1.38 \times 10^{-23}\ J\ K^{-1})(10^8\ K)}{\left(\frac{1}{2}\right)(6.6 \times 10^{-27}\ kg)}$$
$$v^2 = 6.2727 \ldots \times 10^{11}$$
$$v = \sqrt{6.2727 \ldots \times 10^{11}}$$
$$v = 792005.5 \ldots m\ s^{-1}$$
$$v = 7.9 \times 10^5\ m\ s^{-1}\ (2\ s.f.)$$

Nuclear Fission/Fusion and Binding Energy

Exam Style Question 2

(a) Explain what is meant by the statement below.

Radioactivity is a random process.

(b) Uranium-235 was present during the formation of the Solar System, including the Earth. The percentage of the original quantity of ${}^{235}_{92}U$ found in rocks today is 1.1%. The half-life of ${}^{235}_{92}U$ is $7.1 \times 10^8\ years$. Calculate the age, in years, of the Earth.

(c) Fig. 6.1 shows the variation of binding energy per nucleon against nucleon number A.

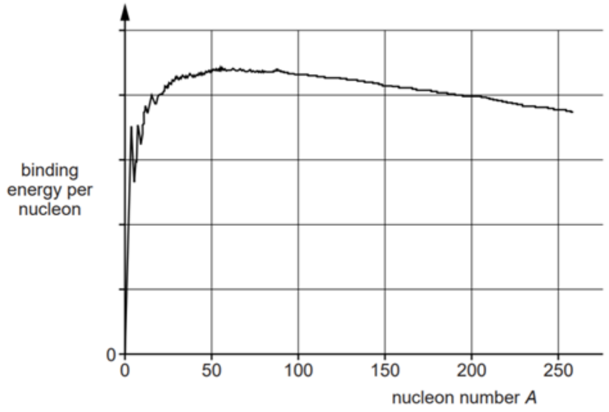


Fig. 6.1

(i) Use Fig. 6.1 to estimate the value of the nucleon number of the most stable isotope.

(ii) Use Fig. 6.1 to explain why nuclei of ${}^{100}_{42}Mo$ cannot produce energy by fusion.

(iii) The mass of a ${}^{8}_{4}Be$ nucleus is $1.329 \times 10^{-26}\ kg$. Use data provided on the second page of the Data, Formulae and Relationships Booklet to determine the binding energy per nucleon for this nucleus.

Nuclear Fission/Fusion and Binding Energy

Exam Style Question 2

(a)Explain what is meant by the statement below.

Radioactivity is a random process.

Impossible to predict when a nucleus will decay or impossible to predict which nucleus will decay.

(b) Calculate the age, in years, of the Earth.

Use $N = N_0 e^{-\lambda t}$ and rearrange for t

Remember we need to calculate the decay constant too:

$$\lambda = \frac{In(2)}{T_{1/2}} = \frac{In(2)}{7.1 \times 10^8\ years}$$

Therefore:

$$0.011 N_0 = N_0 e^{-\left(\frac{In(2)}{7.1 \times 10^8\ y}\right)t}$$

Cancel the N_0

$$0.011 = e^{-\left(\frac{In(2)}{7.1 \times 10^8\ y}\right)t}$$

Rearrange for t

$$In(0.011) = -\left(\frac{In(2)}{7.1 \times 10^8\ y}\right)t$$

$$t = \frac{In(0.011)}{-\left(\frac{In(2)}{7.1 \times 10^8\ y}\right)}$$

$$t = 4619510393\ years$$

Therefore the age of the Earth is 4.6×10^9 years (2 s.f.).

(c) (i) Use Fig. 6.1 to estimate the value of the nucleon number of the most stable isotope.

The most stable nuclei occur around the maximum point on the graph – which is at nucleon number 56 (i.e. Iron, Fe).

(c) (ii) Use Fig. 6.1 to explain why nuclei of ${}^{100}_{42}Mo$ cannot produce energy by fusion.

If Mo was to fuse together it will move away from the maximum average BE per nucleon therefore the BE per nucleon will decrease for fusion which is impossible unless external energy is supplied.

Nuclear Fission/Fusion and Binding Energy

Exam Style Question 2

(a) Explain what is meant by the statement below.

Radioactivity is a random process.

(b) Uranium-235 was present during the formation of the Solar System, including the Earth. The percentage of the original quantity of ${}^{235}_{92}U$ found in rocks today is 1.1%. The half-life of ${}^{235}_{92}U$ is $7.1 \times 10^8\ years$. Calculate the age, in years, of the Earth.

(c) Fig. 6.1 shows the variation of binding energy per nucleon against nucleon number A.

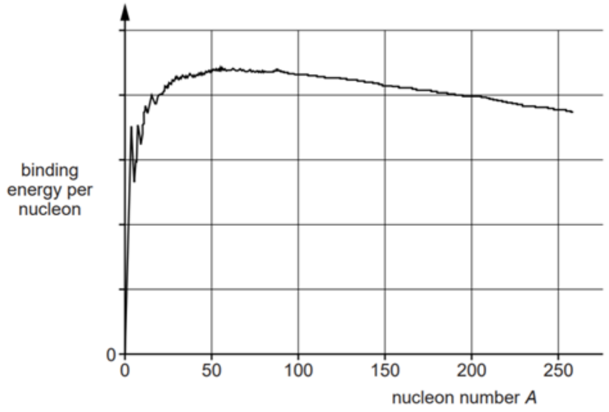


Fig. 6.1

(i) Use Fig. 6.1 to estimate the value of the nucleon number of the most stable isotope.

(ii) Use Fig. 6.1 to explain why nuclei of ${}^{100}_{42}Mo$ cannot produce energy by fusion.

(iii) The mass of a ${}^{8}_{4}Be$ nucleus is $1.329 \times 10^{-26}\ kg$. Use data provided on the second page of the Data, Formulae and Relationships Booklet to determine the binding energy per nucleon for this nucleus.

Nuclear Fission/Fusion and Binding Energy

Exam Style Question 2

(c) (iii) The mass of a $\boldsymbol{{}^{8}_{4}Be}$ nucleus is $\boldsymbol{1.329 \times 10^{-26}\ kg}$. Use data provided on the second page of the Data, Formulae and Relationships Booklet to determine the binding energy per nucleon for this nucleus.

Mass of proton = $1.6726219 \times 10^{-27}\ kg$

Mass of neutron = $1.67493 \times 10^{-27}\ kg$

$\Delta E = \Delta mc^2$ which is also $BE = mass\ defect \times c^2$

$$mass\ of\ nucleons = mass\ of\ protons + mass\ of\ neutrons$$

$$mass\ of\ nucleons = (4 \times 1.673 \times 10^{-27}\ kg) + (4 \times 1.675 \times 10^{-27}\ kg)$$

$$\therefore\ \Delta m = [(4 \times 1.673 \times 10^{-27}\ kg) + (4 \times 1.675 \times 10^{-27}\ kg)] - 1.329 \times 10^{-26}\ kg$$

$$\Delta m = mass\ defect = 1.02 \times 10^{-28}\ kg$$

$$BE = mass\ defect \times c^2$$

$$BE = (1.02 \times 10^{-28}\ kg)(3.0 \times 10^8\ m\ s^{-1})^2$$

$$BE = 9.18 \times 10^{-12}\ J$$

$$\therefore BE\ per\ nucleon = \frac{9.18 \times 10^{-12}\ J}{8}$$

$$BE\ per\ nucleon = 1.15 \times 10^{-12}\ J\ (2\ d.p.)$$

Nuclear Fission/Fusion and Binding Energy

Exam Style Question 3

(a) The following nuclear reaction occurs when a slow-moving neutron is absorbed by an isotope of uranium-235.

$$^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3{}^{1}_{0}n$$

(i) Explain how this reaction is able to produce energy.

(ii) State in what form the energy is released in such a reaction.

(b) The binding energy per nucleon of each isotope in (a) is given in Fig. 8.1.

isotope	binding energy per nucleon / MeV
$^{235}_{92}U$	7.6
$^{141}_{56}Ba$	8.3
$^{92}_{36}Kr$	8.7

Fig. 8.1

(i) Explain why the neutron $^{1}_{0}n$ does not appear in the table above.

(ii) Calculate the energy released in the reaction shown in (a).

Nuclear Fission/Fusion and Binding Energy

Exam Style Question 3

(a) (i) Explain how this reaction is able to produce energy.

In this reaction there is a decrease in the mass and according to $\Delta E = \Delta mc^2$ mass is converted into energy or the total binding energy of the products is greater than the binding energy of the original nucleus and the difference in the binding energies is released as energy.

(a) (ii) State in what form the energy is released in such a reaction.
Kinetic energy

(b) (i) Explain why the neutron $^{1}_{0}n$ does not appear in the table above.
The neutron is a single nucleon and therefore cannot be split further and so no binding occurs.

(b) (ii) Calculate the energy released in the reaction shown in (a).

In this question we are trying to figure out the ΔE:

$$\Delta E = BE\ of\ Uranium - BE\ of\ products$$
$$BE\ of\ Uranium = 235 \times 7.6\ MeV = 1786\ MeV$$
$$BE\ of\ products = BE\ of\ Ba + BE\ of\ Kr$$
$$BE\ of\ products = (141 \times 8.3\ MeV) + (92 \times 8.7\ MeV)$$
$$BE\ of\ products = 1970.7\ MeV$$
$$\therefore energy\ available = 184.7\ MeV$$

Nuclear Fission/Fusion and Binding Energy

Exam Style Question 4

(a) In the core of a nuclear reactor, one of the many fission reactions of the uranium-235 nucleus is shown below.

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2^{1}_{0}n$$

(i) State one quantity that is conserved in this fission reaction.

(ii) Fig. 4.1 illustrates this fission reaction.

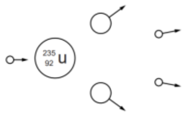


Fig. 4.1

Label all the particles in Fig. 4.1 and extend the diagram to show how a chain reaction might develop.

(b) Fusion of hydrogen nuclei is the source of energy in most stars. A typical reaction is shown below.

$$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + ^{1}_{0}n$$

The $^{2}_{1}H$ nuclei repel each other. Fusion requires the $^{2}_{1}H$ nuclei to get very close and this usually occurs at very high temperatures, typically $10^{9}\ K$.

(i) Use the data below to calculate the energy released in the fusion reaction above.

- mass of $^{2}_{1}H$ nucleus = $3.343 \times 10^{-27}\ kg$
- mass of $^{3}_{2}He$ nucleus = $5.006 \times 10^{-27}\ kg$
- mass of $^{1}_{0}n$ = $1.675 \times 10^{-27}\ kg$

(ii) State in what form the energy in (b)(i) is released.

(iii) The $^{2}_{1}H$ nuclei in stars can be modelled as an ideal gas. Calculate the mean kinetic energy of the $^{2}_{1}H$ nuclei at $10^{9}\ K$.

(iv) Suggest why some fusion can occur at a temperature as low as $10^{7}\ K$.

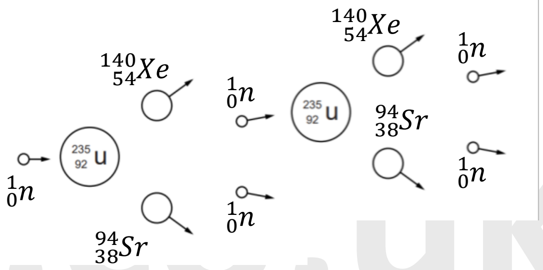
Nuclear Fission/Fusion and Binding Energy

Exam Style Question 4

(a) (i) State one quantity that is conserved in this fission reaction.

- Momentum
- Charge
- Proton number
- Baryon number
- Nucleon number

(a) (ii) Label all the particles in Fig. 4.1 and extend the diagram to show how a chain reaction might develop.



(b) (i) Use the data below to calculate the energy released in the fusion reaction above.

Use $\Delta E = \Delta mc^2$

$$initial\ mass = ^{2}_{1}H + ^{2}_{1}H = (3.343 \times 10^{-27} kg) + (3.343 \times 10^{-27}\ kg)$$
$$initial\ mass = 6.686 \times 10^{-27}\ kg$$
$$final\ mass = ^{3}_{2}He + ^{1}_{0}n = (5.006 \times 10^{-27}\ kg) + (1.675 \times 10^{-27}\ kg)$$
$$final\ mass = 6.681 \times 10^{-27}\ kg$$
$$\Delta m = initial\ mass - final\ mass$$
$$\Delta m = (6.686 \times 10^{-27}\ kg) - (6.681 \times 10^{-27}\ kg) = 5 \times 10^{-30}\ kg$$
$$\therefore\ \Delta E = (5 \times 10^{-30}\ kg)(3 \times 10^{8})^{2}$$
$$\Delta E = 4.5 \times 10^{-13}\ J$$

(b) (ii) State in what form the energy in (b)(i) is released.

Kinetic energy.

Nuclear Fission/Fusion and Binding Energy

Exam Style Question 4

(a) In the core of a nuclear reactor, one of the many fission reactions of the uranium-235 nucleus is shown below.

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2^{1}_{0}n$$

(i) State one quantity that is conserved in this fission reaction.

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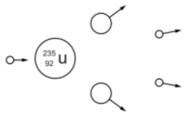


Fig. 4.1

Label all the particles in Fig. 4.1 and extend the diagram to show how a chain reaction might develop.

(b) Fusion of hydrogen nuclei is the source of energy in most stars. A typical reaction is shown below.

$$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + ^{1}_{0}n$$

The $^{2}_{1}H$ nuclei repel each other. Fusion requires the $^{2}_{1}H$ nuclei to get very close and this usually occurs at very high temperatures, typically $10^9\ K$.

(i) Use the data below to calculate the energy released in the fusion reaction above.

- mass of $^{2}_{1}H$ nucleus = $3.343 \times 10^{-27}\ kg$
- mass of $^{3}_{2}He$ nucleus = $5.006 \times 10^{-27}\ kg$
- mass of $^{1}_{0}n$ = $1.675 \times 10^{-27}\ kg$

(ii) State in what form the energy in (b)(i) is released.

(iii) The $^{2}_{1}H$ nuclei in stars can be modelled as an ideal gas. Calculate the mean kinetic energy of the $^{2}_{1}H$ nuclei at $10^9\ K$.

(iv) Suggest why some fusion can occur at a temperature as low as $10^7\ K$.

Nuclear Fission/Fusion and Binding Energy

Exam Style Question 4

(b) (iii) Calculate the mean kinetic energy of the $^{2}_{1}H$ nuclei at $10^9\ K$.

Use $KE = \frac{3}{2}kT$

$$KE = \frac{3}{2}(1.38 \times 10^{-23}\ J\ K^{-1})(10^9\ K)$$
$$KE = 2.1 \times 10^{-14}\ J\ (2\ s.f.)$$

(b) (iv) Suggest why some fusion can occur at a temperature as low as $10^7\ K$.

Some nuclei will have KE greater than the mean KE and hence cause fusion.

Nuclear Fission/Fusion and Binding Energy

Exam Style Question 5

The nuclear reaction represented by the equation

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{94}_{39}Y + ^{139}_{53}I + 3^{1}_{0}n$$

takes place in the core of a nuclear reactor at a power station.

(a) Describe how this reaction can lead to a chain reaction.

(b) Explain the role of fuel rods, control rods and a moderator in a nuclear reactor.

(c) In the nuclear reactor of a power station, each fission reaction of uranium produces $3.2 \times 10^{-11}\ J$ of energy. The electrical power output of the power station is $3.0\ GW$. The efficiency of the system that transforms nuclear energy into electrical energy is 22%. Calculate

(i) the total power output of the reactor core

(ii) the total energy output of the reactor core in one day

$1\ day = 8.64 \times 10^4\ s$

(iii) the mass of uranium-235 converted in one day. The mass of a uranium-235 nucleus is $3.9 \times 10^{-25}\ kg$.

(d) Discuss the physical properties of nuclear waste that makes it dangerous.

Nuclear Fission/Fusion and Binding Energy

Exam Style Question 5

(a) Describe how this reaction can lead to a chain reaction.
The three neutrons produced interact with other uranium nuclei with cause further fission reactions.

(b) Explain the role of fuel rods, control rods and a moderator in a nuclear reactor.
Fuel rod: Contain the uranium or fissile material.

Control rods: Absorb some of the neutrons.
In a controlled chain reaction the control rods are inserted into the reactor so as to allow on average one neutron from previous reaction to cause subsequent fission.

Moderator: Slows down the fast-moving neutrons and lowers the KE of neutrons. Slow moving neutrons have a greater chance of causing fission as they are more likely to be absorbed by the U-235 and therefore sustaining chain reaction.

(c) Calculate
(i) the total power output of the reactor core.
We know the electrical power output of the power station is $3.0\ GW$ and that is only 22% of the power station therefore the total power output of the reactor core is:

$$total\ power\ output = \frac{3.0 \times 10^9\ W}{0.22}$$
$$total\ power\ output = 1.36 \times 10^{10}\ W\ (2\ d.p.)$$

(ii) the total energy output of the reactor core in one day.
Use $P = \frac{E}{t}$ and rearrange for E

$$E = P \times t = (1.36 \times 10^{10}\ W)(8.64 \times 10^4\ s)$$
$$E = 1.18 \times 10^{15}\ J\ (2\ d.p.)$$

(iii) the mass of uranium-235 converted in one day. The mass of a uranium-235 nucleus is $\mathbf{3.9 \times 10^{-25}\ kg}$.

$$no.of\ reactions\ per\ day = \frac{1.18 \times 10^{15}\ J}{3.2 \times 10^{-11}\ J} = 3.6875 \times 10^{25}$$

1 reaction equals 1 uranium therefore:

$$mass\ of\ U - 235 = (3.6875 \times 10^{25})(3.9 \times 10^{-25}\ kg)$$
$$mass\ of\ U - 235 = 14.4\ kg\ (2\ d.p.)$$

(d) Discuss the physical properties of nuclear waste that makes it dangerous.
Nuclear waste is radioactive for a long time and can cause ionisation.

Nuclear Fission/Fusion and Binding Energy

Exam Style Question 6

(a) Describe the process of induced nuclear fission.

(b) Explain how nuclear fission can provide energy.

(c) Suggest a suitable material which can be used as a moderator in a fission reactor and explain its role.

Nuclear Fission/Fusion and Binding Energy

Exam Style Question 6

(a) Describe the process of induced nuclear fission.
A neutron is absorbed by a massive uranium nucleus. The nucleus splits into two smaller nuclei and one or more neutrons are released.

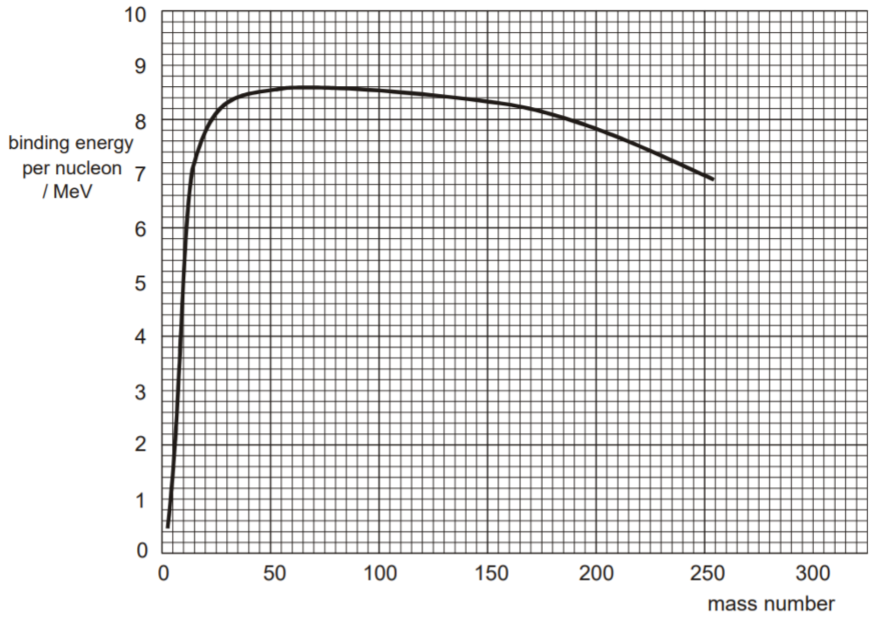
(b) Explain how nuclear fission can provide energy.
In a fission reaction there is a decrease in the mass. According to $\Delta E = \Delta mc^2$ mass is converted into energy.

(c) Suggest a suitable material which can be used as a moderator in a fission reactor and explain its role.
Water, graphite or carbon can be used as a moderator.
A moderator slows down the fast moving neutrons and reduces the KE of neutrons. This is because the slow-moving neutrons have a greater chance of causing fission than fast-moving neutrons.

Nuclear Fission/Fusion and Binding Energy

Exam Style Question 7

The figure below shows the variation with nucleon number (mass number) of the binding energy per nucleon for various nuclides.



(a) (i) State the number of nucleons in the nucleus of ${}^{94}_{37}Rb$.

(ii) State the number of protons in the nucleus of ${}^{142}_{55}Cs$.

(iii) State the number of neutrons in the nucleus of ${}^{235}_{92}U$.

(b) Use the figure above to calculate the energy released when a ${}^{235}_{92}U$ nucleus undergoes fission, producing nuclei of ${}^{94}_{37}Rb$ and ${}^{142}_{55}Cs$.

Nuclear Fission/Fusion and Binding Energy

Exam Style Question 7

(a)(i) State the number of nucleons in the nucleus of ${}^{94}_{37}Rb$.

Nucleons = 94

(ii) State the number of protons in the nucleus of ${}^{142}_{55}Cs$.

Protons = 55

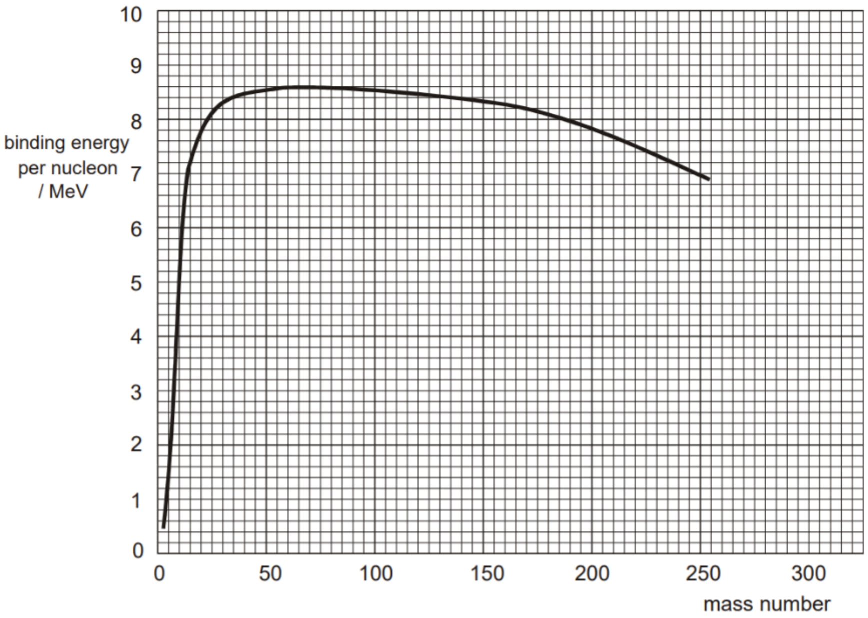
(iii) State the number of neutrons in the nucleus of ${}^{235}_{92}U$.

Neutrons = 235-92 = 143

Nuclear Fission/Fusion and Binding Energy

Exam Style Question 7

The figure below shows the variation with nucleon number (mass number) of the binding energy per nucleon for various nuclides.



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(ii) State the number of protons in the nucleus of ${}^{142}_{55}Cs$.

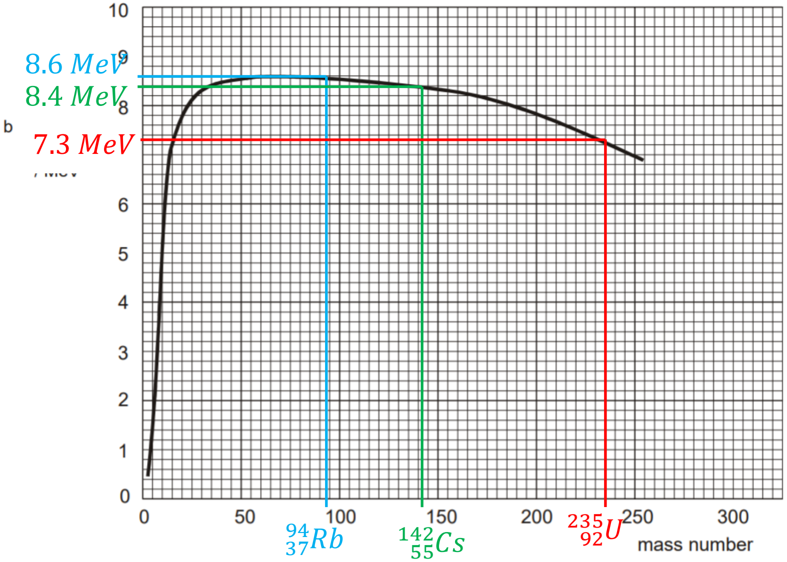
(iii) State the number of neutrons in the nucleus of ${}^{235}_{92}U$.

(b) Use the figure above to calculate the energy released when a ${}^{235}_{92}U$ nucleus undergoes fission, producing nuclei of ${}^{94}_{37}Rb$ and ${}^{142}_{55}Cs$.

Nuclear Fission/Fusion and Binding Energy

Exam Style Question 7

(b) Use the figure above to calculate the energy released when a ${}^{235}_{92}U$ nucleus undergoes fission, producing nuclei of ${}^{94}_{37}Rb$ and ${}^{142}_{55}Cs$.



Therefore the binding energy for:

$$U = 7.3\ MeV$$
$$Cs = 8.4\ MeV$$
$$Rb = 8.6\ MeV$$

So the total binding energy is:

$$U = 235 \times 7.3\ MeV = 1715.5\ MeV$$
$$Cs = 142 \times 8.4\ MeV = 1192.8\ MeV$$
$$Rb = 94 \times 8.6\ MeV = 808.4\ MeV$$

Total binding energy:

$$E = total\ BE\ for\ products - BE\ of\ U$$
$$E = (1192.8\ MeV + 808.4\ MeV) - 1715.5\ MeV$$
$$E = 285.7\ MeV$$

Nuclear Fission/Fusion and Binding Energy

Exam Style Question 8

(a) (i) State two physical features or properties required of the shielding to be placed around the reactor at a nuclear power station.

(ii) Which material is usually used for this purpose?

(b) Describe the effect of the shielding on the γ rays, neutrons and neutrinos that reach it from the core of the reactor. Also explain why the shielding material becomes radioactive as the reactor ages.

Nuclear Fission/Fusion and Binding Energy

Exam Style Question 8

(a)(i) State two physical features or properties required of the shielding to be placed around the reactor at a nuclear power station.

- Thick
- High density
- Material giving minimal fatigue problems after irradiation.

(ii) Which material is usually used for this purpose?
Reinforced concrete.

(b) Describe the effect of the shielding on the γ rays, neutrons and neutrinos that reach it from the core of the reactor. Also explain why the shielding material becomes radioactive as the reactor ages.

Effect of shielding:
γ-rays intensity is greatly reduced.
Some neutrons are absorbed.
Very little effect on neutrinos.

Why shielding becomes radioactive:
Neutron absorption by nuclei or atoms of the shielding makes the nuclei neutron rich and unstable thereby the shielding becoming β^- emitters or γ emitters.

Please see '10.2.1 Nuclear Fission and Fusion notes' pack for revision notes.

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