



AS Level Physics

Chapter 1 – Measurements and their errors

1.2.1 Measurements and Uncertainties

Notes

Variables

A variable is a quantity whose value can change, such as mass. Variables are classified into two categories:

- Independent variable: a quantity that can be changed in an experiment.
- Dependent variable: a quantity that can be measured in an experiment.

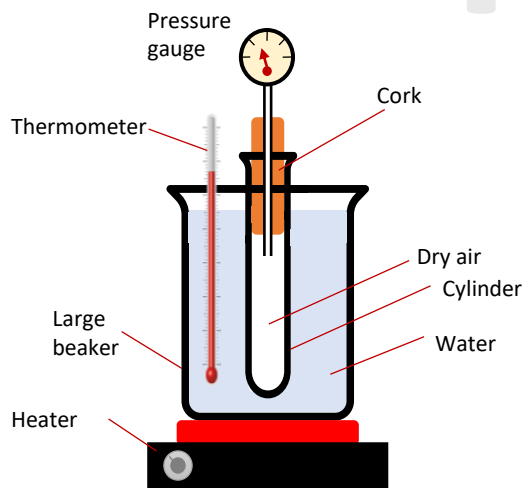
To ensure that the experiment is a fair test, all other variables must be kept constant.

This means that any effects you observe during the experiment are due to the independent variable being changed.

Control variables are the variables in an experiment that are kept constant.

Example:

You could set up the experiment presented below to investigate the effect of temperature on pressure of a fixed mass of gas at constant volume.



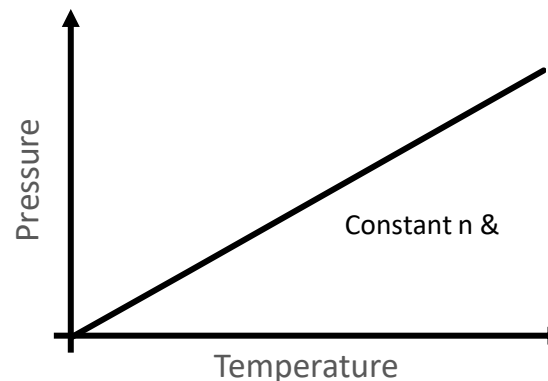
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Variables

Example continued:

- The independent variable will be the temperature of the water surrounding the cylinder.
- The dependent variable will be the pressure of the gas in the cylinder.
- All other variables must be kept the same. These include the volume inside the cylinder, V (done by trapping air using a cork), the type of gas used and the mass of the gas.

When drawing graphs, the dependent variable should be on the (vertical) y-axis, and the independent variable on the (horizontal) x-axis.



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Types of Data

There are various sorts of data, including:

1) Discrete data

Discrete data is data which can only take certain values. E.g. number of students in a class. You can't have 1.5 students in a classroom. Shoe size is another example of a discrete variable – only certain values are allowed such as 5, 5.5, 6 and can't have 6.75.

Discrete data is commonly presented in scatter graphs and bar charts.

2) Continuous data

A continuous variable can have any value on a scale. E.g. temperature. You can never measure the exact value of a continuous variable.

The common way to display two sets of continuous data is a line graph or a scatter graph.

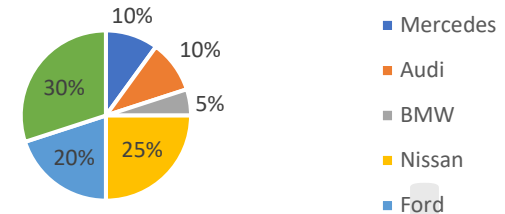
Types of Data

3) Categorical data

A categorical variable has values that can be sorted into categories. E.g. type of hair colour can be black, brown, blonde, etc ...

Categorical data is commonly presented on a pie chart or a bar chart.

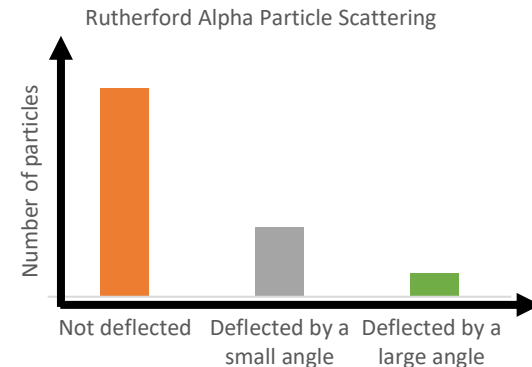
Categorical data of different types of car manufacturers in a particular country



4) Ordered (ordinal) data

Ordered data is like categorical data, but the categories are in a particular order. E.g. if you classified chillies based on spiciness as 'mild', 'hot' and 'very hot' you will have ordered data.

Ordered data is commonly presented on a bar chart.



Errors

When conducting experiments, errors can occur, which can mean the final results are not always the true values.

There are many sources of error but they all can be categorised into:

- **Systematic error** – can be caused by faulty equipment, the environment, or a bad experimental method (e.g. using an inaccurate clock).
- **Random error** – caused by unpredictable changes in the experiment. You get random errors in any measurement. These include human error (i.e. you can measure a length of string 10 times and you'll probably get slightly different values each time).

Systematic Error

Systematic errors result in true values to be different by the same amount each time. Systematic errors can be caused by:

- Zero error: which is when an instrument is not correctly zeroed. E.g. a scale that does not read zero when no object is placed upon it. This means all readings are going to be higher than the true value. Either correct the scale to read zero, or adjust all readings to take account of the error.
- An incorrect calibration of an instrument: e.g. an improperly calibrated ammeter that shows a value that is higher or lower than the true value, e.g. it may read 4.9A when 5A is flowing.
- Incorrect use of an instrument: e.g. a Bunsen burner on a orange flame when it should be on the blue flame. Learn the correct technique for using instruments and apparatus.

Error

- Parallax error – occurs when a person reads an instrument at an angle, resulting in a reading that is either consistently high or consistently low.

It is possible to reduce or even eliminate systematic error.

If systematic error is suspected, repeat the experiment using a different technique or apparatus and compare the results.

You can also calibrate your device by taking a measurement of a known value (i.e. a known mass). If the measured and known values differ, you can use the difference to adjust the apparatus's inaccuracy and therefore lower your systematic error.

Calibration can also reduce zero errors.

Random error

Random errors are unpredictable and can be due to human error.

Random errors occur as a result of the experimenter's judgements:

- Human reaction: i.e. timing a moving object. When did it first start moving? What time did it cross the finish line? You make that judgement.
- Reading from a scale: You may have to determine where a meter needle is on a scale.

Random errors can also occur because the circumstances under which the measurements are taken can differ, e.g.:

- Conditions can vary: changes in room temperature or humidity can have an impact on the results.
- Equipment can vary: one trolley may have more friction than another.
- Samples of materials may be different: The components of two springs from the same reel may differ slightly.



Errors

Some measures are inherently random:

- Radioactive decay: If you monitor background radiation in the lab for 30 seconds, you'll probably get somewhat different results each time.

Random errors can be decreased, but they are almost never completely eliminated. Here are some suggestions for reducing random error:

- Make multiple measurements, and find the mean (average). Normally, you'd take three or four reads before calculating the mean.
- Plot a graph, and draw a smooth curve or a straight line through the points to find a correlation.
- To reduce errors of judgement, use a suitable device, such as light gates and an electronic timer instead of a stopwatch.

Precision and Accuracy

Precision and accuracy are not the same.

Precision: is how close together repeat values are – the lower the spread, the more precise the data.

The precision is improved by reducing random errors.

Accuracy: is a measure of how close a numerical result is to the true value. As a result, if a measure is close to the true value, it is accurate.

The accuracy of the final result is improved by reducing systematic errors.



Uncertainties

Uncertainties can be represented as absolute amounts, fractions and percentages.

- Absolute uncertainty refers to uncertainty of a measurement expressed as a fixed number.
- Fractional uncertainty refers to uncertainty expressed as a fraction of the measurement taken.
- Percentage uncertainty refers to uncertainty expressed as a percentage of the measurement.

A level of confidence could also be included in an uncertainty to indicate how likely the true value is to lie inside the interval. E.g. '5.0 ± 0.4' indicates that the true value is somewhere between 4.6 and 5.4.

Example:

A bulb's resistance is specified as $6.0 \pm 0.5\Omega$. Give the absolute, fractional and percentage uncertainties for this measurement.

- The absolute uncertainty is given in the question - 0.5Ω
- To calculate fractional uncertainty, divide the uncertainty by the measurement and simplify:

The fractional uncertainty is $\frac{0.5}{6.0} = \frac{1}{12}$

- To calculate percentage uncertainty, divide the uncertainty by the measurement and multiply by 100:

The percentage uncertainty is $\frac{1}{12} \times 100 = 8.3\%$

Uncertainties

Examples

- If there is only one reading, or all repeat readings are the same, the absolute uncertainty is \pm smallest increment, for example:

Width of wire measured with a ruler (precision 1mm): 24mm, 24mm, 24mm

Width = $24 \pm 1\text{mm}$

- If the repeat readings are different, the absolute uncertainty is $\pm \left(\frac{\text{range}}{2}\right)$, e.g.:

Width of stick measured with vernier calliper (precision 0.1mm):

24.1mm, 24.5mm, 24.3mm

Width = $24.3 \pm 0.20\text{mm}$

- If the two measurements are added or subtracted, the absolute uncertainties are added:

Mass of rocket before launch = $400 \pm 0.5\text{g}$

Mass of rocket after launch = $105 \pm 0.5\text{g}$

Mass lost = $400 - 105 = 295 \pm 1\text{g}$



Uncertainties

Example

- The percentage uncertainty does not change when multiplying or dividing by a constant number.

$$\text{Radius} = 6\text{mm} \pm 8\%$$

$$\text{Diameter} = \text{radius} \times 2 = 12\text{mm} \pm 8\%$$

- Unit conversions do not change percentage uncertainty.

$$\text{Diameter} = 12\text{mm} \pm 8\%$$

$$\text{Diameter} = 1.2\text{cm} \pm 8\%$$

$$\text{Diameter} = 0.012\text{m} \pm 8\%$$

- The percentage uncertainties are added when multiplying or dividing two measured quantities.

$$\text{Width} = 20\text{mm} \pm 10\%$$

$$\text{Length} = 55\text{mm} \pm 5\%$$

$$\text{Area} = 20\text{mm} \times 55\text{mm} = 1100\text{mm}^2 \pm 15\%$$

- Raising a measurement to the power n multiplies the percentage uncertainty by n .

$$\text{Radius} = 6\text{mm} \pm 10\%$$

$$\text{Area} = \pi \times \text{radius}^2$$

$$\text{Area} = 113\text{mm}^2 \pm 20\%$$



Uncertainties

Don't forget to convert all the uncertainties to percentages before you combine by multiplying or dividing.

Example

The linear velocity of a passenger car on a Ferris wheel is $2.5 \pm 3\% \text{ms}^{-1}$. The Ferris wheel has a radius of $35 \pm 2\text{m}$. Determine the angular velocity of the passenger car and state the percentage uncertainty in this value.

- 1) First calculate the angular velocity without uncertainty:

$$\omega = \frac{v}{r} = \frac{2.5}{35} = 0.071 \text{ rad s}^{-1}$$

- 2) Next calculate the percentage uncertainty in the radius:

$$\% \text{ uncertainty in } r = \frac{2}{35} \times 100 = 5.71\%$$

- 3) To determine the total uncertainty in the angular velocity, add the percentage uncertainties in the velocity and radius values:

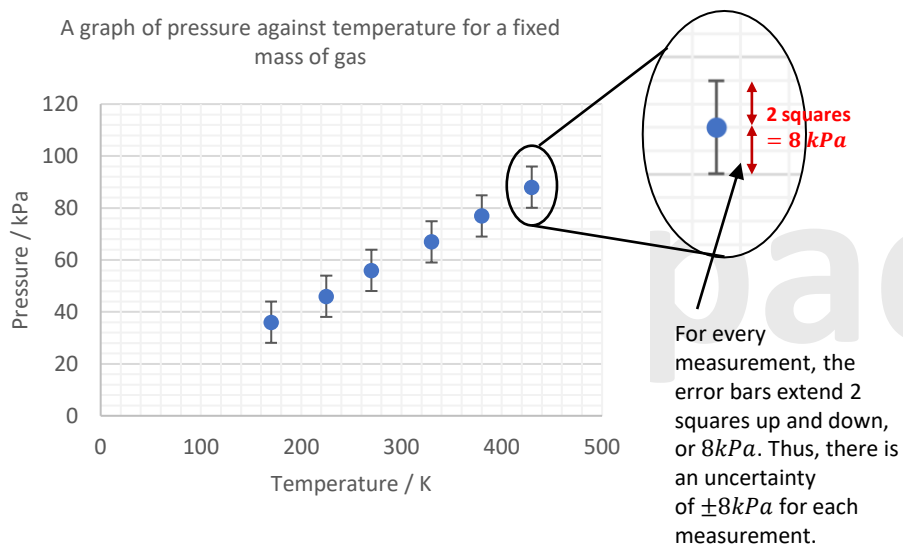
$$\text{Total uncertainty} = 3 + 5.71 = 8.71\%$$

$$\text{So, the angular velocity} = 0.071 \pm 8.71\% \text{ rad s}^{-1}$$

Error Bars

Usually and from our previous examples, you calculate the uncertainty in the final result using the uncertainty in each measurement you record. When plotting a graph, you show the uncertainty in each measurement by using error bars to indicate the range in which the point is most likely to fall.

The error bars in the graph below can be used to determine the error in measuring the pressure of a gas container.



The line of best fit is a line that runs through a scatter plot of data points and best reflects the relationship between them. The line of best fit passes through all error bars and has an even number of points above and below the line.



Error Bars

The line of worst fit are those that could potentially fit the data if the "actual" value for each point could lie anywhere within the error bars' range. The line of worst fit goes through the error bars but has a maximum or minimum possible gradient. The line of worst fit matches the trend as badly as possible while still passing through the error bars. Lines of worst fit don't really have an even number of points above or below the line.

Even though your line of best fit and line of worst fit should go through all the errors anomalous data should be excluded.

See the next page for an example of the line of best and worst fit.

Percentage uncertainty in gradient

The percentage uncertainty in the gradient is obtained by calculating the gradient for both the best fit and worst fit line.

You use the following formula to calculate the gradient:

$$\text{gradient} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Then, once you've calculated the gradient for both lines, use:

$$\% \text{ uncertainty} = \frac{\text{gradient of best fit} - \text{gradient of worst fit}}{\text{gradient of best fit}} \times 100$$

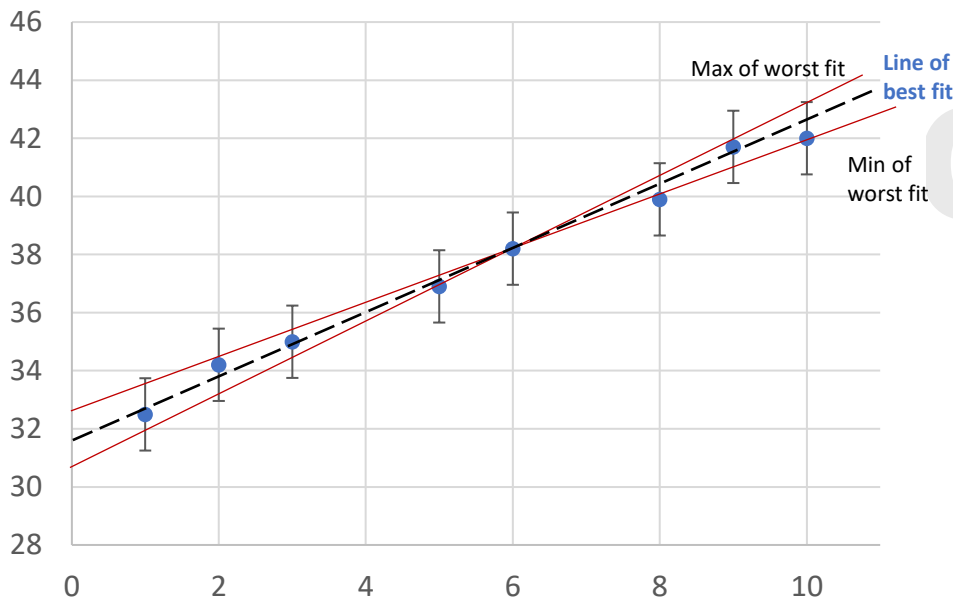
Error Bars

Percentage uncertainty in y-intercept

To calculate the percentage uncertainty in the y-intercept you will need to plot three lines – the line of best fit and the maximum and minimum worst fit lines.

Each line's y-intercept is the point at which it crosses the y-axis. Then use the following formula to calculate the percentage uncertainty:

$$\% \text{ uncertainty} = \frac{\text{max y intercept} - \text{min y intercept}}{\text{line of best fit y intercept}} \times 100$$



Evaluations

Now that you know how to calculate uncertainty, you'll need to assess your findings to see how credible they are. Valid, accurate, precise, repeatable, and reproducible are all words that sound similar, but they all mean different things when it comes to your results.

1) Precise result

Is how close together repeat values are – the lower the spread, the more precise the data. The amount of random error in your readings determines precision.

2) Repeatable results

If you can repeat an experiment numerous times and achieve the same results, the results are repeatable.

3) Reproducible results

If someone else can repeat your experiment with different equipment or methods and obtain the same results as you, the results are reproducible.

4) Valid results

A valid result arises from a suitable experimental method and how appropriate it is in answering the experiments original question. In order to obtain valid results, all variables other than the ones you're investigating should be kept constant. You haven't actually tested the variables under investigation if those variables aren't kept constant, making the results invalid.

5) Accurate results

An accurate result is a measure of how close a numerical result is to the true value. As a result, if a measure is close to the true value, it is accurate. Only knowing the true value of a result allows you to comment on its accuracy.



Evaluations

Remember that results can be precise but not accurate, e.g. a balance that weighs to $1/100^{\text{th}}$ of a gram is likely to give precise results, but the results will not be accurate if it is not calibrated correctly.

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