AS Level Physics

Chapter 4 – Materials

4.1.1 Density and Upthrust

Notes

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DENSITY

The density of a substance is the mass per unit volume:

$$\boldsymbol{\rho = \frac{m}{V}}$$

Where:

- $\boldsymbol{\rho} = density\ measured\ in\ kgm^{-3}$
- $\boldsymbol{m} = mass\ measured\ in\ kg$
- $\boldsymbol{V} = volume\ measured\ in\ m^3$

Density has a symbol, ρ (rho).

The more mass in a given volume, the greater the density.

DENSITY

Example 1:

A rectangular block of steel measures 10 cm x 7.5 cm x 5 cm and has a mass of 9 kg. Calculate the density of the steel.

1) Calculate the volume of the rectangular block:

$$V = 0.1\ m \times 0.075\ m \times 0.05\ m$$

$$V = 3.75 \times 10^{-4}\ m^3$$

2) We know the mass to be 9 kg so use:

$$\rho = \frac{m}{V}$$

$$\rho = \frac{9\ kg}{3.75 \times 10^{-4}\ m^3}$$

$$\boldsymbol{\rho = 24000\ kgm^{-3}}$$

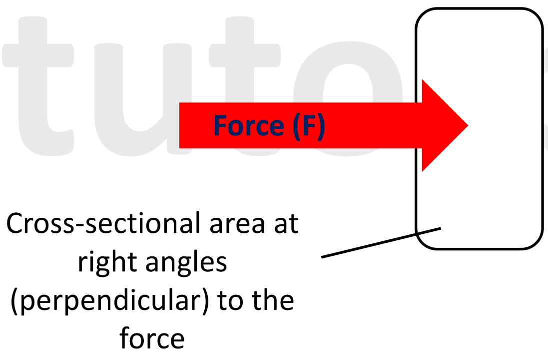
PRESSURE

The pressure is defined as the perpendicular force per unit area.

$$P = \frac{F}{A}$$

Where:

- $\boldsymbol{P} = pressure\ measured\ in\ Pascals\ (Pa)$
- $\boldsymbol{F} = perpendicular\ force\ measured\ in\ Newtons\ (N)$
- $\boldsymbol{A} = cross - sectional\ area\ measured\ in\ metres\ squared\ (m^2)$



The unit of pressure is the Pascal (Pa).

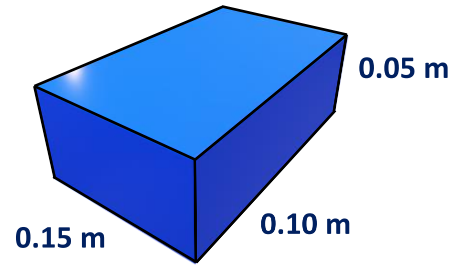
Alternative units for pressure are $\boldsymbol{Newton\ per\ metre^2}$:

$$\boldsymbol{1\ Pa = 1\ Nm^{-2}}$$

PRESSURE

Example 1:

A 5 kg block has dimensions $0.15\ m \times 0.10\ m \times 0.05\ m$. Calculate the pressure the block exerts on the surface of a desk.



1) The only force that is being exerted on the surface of the desk is caused by the weight of the block. The weight force is acting perpendicularly to the desk:

$$W = mg$$
$$W = 5\ kg \times 9.81\ ms^{-2}$$
$$W = 49.05\ N$$

2) Calculate the cross-sectional area. The only area you should calculate and use is the one that is in contact with the desk. In this case its:

$$A = 0.15\ m \times 0.10\ m$$
$$A = 0.015\ m^2$$

3) Calculate the pressure:

$$P = \frac{49.05\ N}{0.015\ m^2}$$

$$\boldsymbol{P = 3270\ Pa}$$

ATMOSPHERIC PRESSURE

Pressure is exerted in all directions at the Earths surface due to air. This is due to air molecules travelling in rapid random motion. When these air molecules collide with the surfaces they exert a force.

The force exerted by the atmosphere is known as the atmospheric pressure with a value of:

$$1.01 \times 10^5 \ Pa$$

At higher altitudes atmospheric pressure decreases as there are fewer air molecules present and less weight is pushing them down from above.

PRESSURE IN LIQUIDS

At the surface of a liquid the pressure is the same as atmospheric pressure.

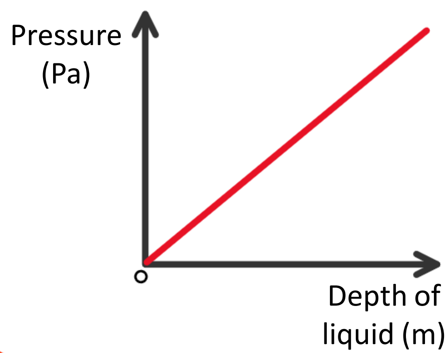
However, below the surface of a liquid, the pressure varies according to the depth and density of the liquid. This can be shown using the formula below:

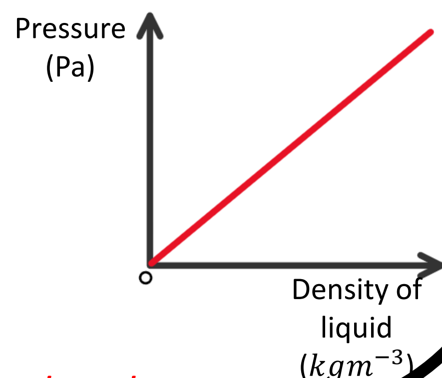
$$P = h\rho g$$

Where:

- $P = pressured\ measured\ in\ Pa$
- $h = depth\ measured\ in\ m$
- $\rho = density\ measured\ in\ kgm^{-3}$
- $g = gravitational\ field\ strength\ measured\ in\ Nkg^{-1}$

As the depth increases. So does the pressure this is due to the weight of all the water above pushing down on the water below. The graphs below shows this relationship:





PRESSURE IN LIQUIDS

Example 1:

An object at 7 m is placed in a liquid with a density of $900\ kgm^{-3}$. Calculate the pressure due to liquid.

$$P = h\rho g$$

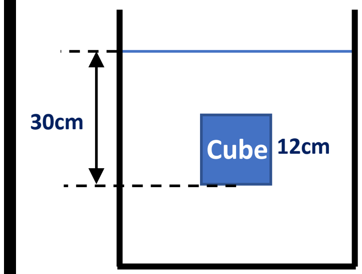
$$P = (7\ m)(900\ kgm^{-3})(9.81\ ms^{-2})$$

$$\boldsymbol{P = 61803\ Pa}$$

PRESSURE IN LIQUIDS

Example 2:

A cube with sides 12 cm is submerged in water to a depth of 30cm.



The density of fresh water is $1 \times 10^3 kgm^{-3}$

(a) Calculate the pressure at the bottom surface of the cube due to the water.

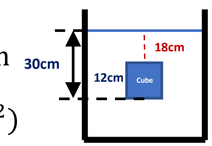
$$Use: P = h\rho g$$

$$P = (0.3m)(1 \times 10^3\ kgm^{-3})(9.81\ ms^{-2})$$

$$\boldsymbol{P = 2940\ Pa}$$

(b) Calculate the pressure at the top surface of the cube due to the water.

Calculate the depth first: $h = 30 - 12 = 18\text{cm} = 0.18\text{m}$



$$Use: P = h\rho g$$

$$P = (0.18m)(1 \times 10^3\ kgm^{-3})(9.81\ ms^{-2})$$

$$\boldsymbol{P = 1764\ Pa}$$

(c) Calculate the force acting on the bottom surface of the cube.

$$Use: Force = Pressure \times Area$$

$$Force = (2940\ Pa)(0.12 \times 0.12)$$

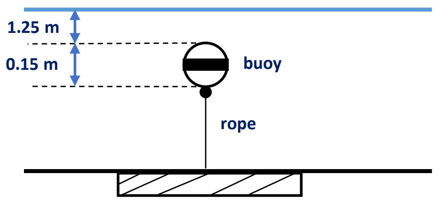
$$\boldsymbol{Force = 42.3\ N}$$

The area of a cube is length x width = 0.12 m x 0.12 m

PRESSURE IN LIQUIDS

Example 3:

A mooring buoy is tethered to the bottom of a sea water loch by a vertical cable as shown:



The density of fresh water is $1.02 \times 10^3 kgm^{-3}$

a) Calculate the total pressure on the top of the buoy:

$$Use: P = h\rho g$$
$$P = (1.25m)(1.02 \times 10^3\ kgm^{-3})(9.81\ ms^{-2})$$
$$\mathbf{P = 12507.75\ Pa}$$

This pressure is only due to the sea water.

To find the total pressure, we need to add the atmospheric pressure from the air above.

$$P_{total} = 12507.75\ Pa + (1.01 \times 10^5\ Pa)$$
$$P_{total} = 113507.75\ Pa$$
$$\mathbf{P_{total} = 1.14 \times 10^5\ Pa}$$

PRESSURE IN LIQUIDS

Example 3:

b) Calculate the total pressure on the bottom of the buoy:

The depth $h = 1.25m + 0.15m = 1.4m$

$$Use: P = h\rho g$$
$$P = (1.4m)(1.02 \times 10^3\ kgm^{-3})(9.81\ ms^{-2})$$
$$\mathbf{P = 14008.68\ Pa}$$

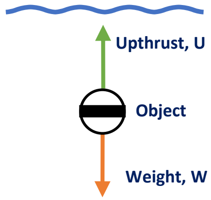
This pressure is only due to the sea water.

To find the total pressure, we need to add the atmospheric pressure from the air above.

$$P_{total} = 14008.68 + (1.01 \times 10^5)$$
$$P_{total} = 115{,}008.68\ Pa$$
$$\mathbf{P_{total} = 1.15 \times 10^5\ Pa}$$

UPTHRUST

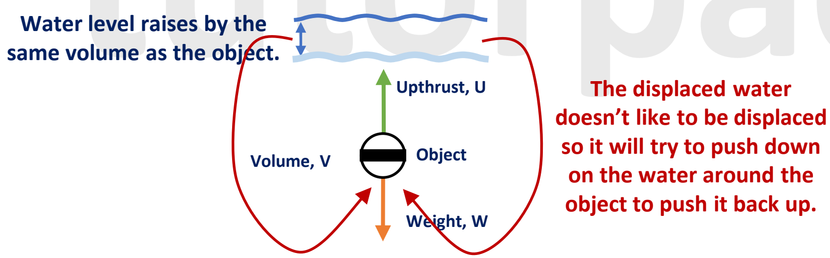
Upthrust is an upwards force which is caused by the fluid pressure when an object is submerged in a fluid.



According to the Archimedes' principle the upthrust exerted on an object is equal to the weight of the fluid it displaces.

For example:

An object submerged in a fluid will experience two forces: weight (due to gravity) acting downwards and upthrust (upwards force). Upthrust is always present, but lets look at why this is the case?



Upthrust is present because when an object with volume, V, is submerged, the water level raises by the same volume as the object. Raising the water level means the water is now displaced however water doesn't like to be displaced and so it will try to push down on the water around the object to try push it back up. The weight of the water that's been displaced is equal to the upthrust the object experiences. Therefore:

$$Upthrust = weight\ of\ fluid\ displaced$$

UPTHRUST

To understand the origin of upthrust, we will use in the next example:

$$\boldsymbol{P = h\rho g}$$

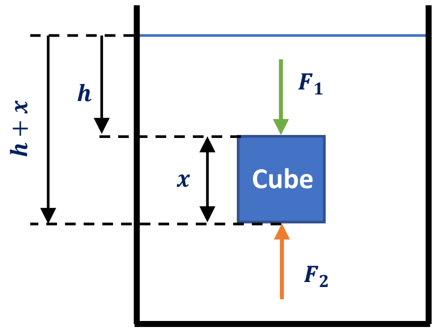
Where:

- $\boldsymbol{P} = pressured\ measured\ in\ Pa$
- $\boldsymbol{h} = depth\ measured\ in\ m$
- $\boldsymbol{\rho} = density\ measured\ in\ kgm^{-3}$
- $\boldsymbol{g} = gravitional\ field\ strength\ measured\ in\ Nkg^{-1}$

We looked at this equation a few pages back.

UPTHRUST

Lets take a block with sides x cm is submerged in water to a depth of h. Two forces act on the object: F_1 and F_2.



As $P = \frac{F}{A}$ (where $P = pressure\ due\ to\ the\ fluid$ and $A = the\ cross-sectional\ area\ that\ the\ force\ acts\ on$) we can rearrange to get force, F :

$$F = PA \ \ldots\ldots\ldots(1)$$

Also, pressure at any depth is equal to

$$\boldsymbol{P = h\rho g} \ \ldots\ldots\ldots(\mathbf{2})$$

So combining equations (1) and (2) we get F_1 to be:

$$F_1 = h\rho g A$$

F_2 acts on the bottom surface of the block which is at a lower depth due to the size of the block (x) therefore:

$$F_2 = \rho g(h + x)A$$

UPTHRUST

So the resultant force (F_R) is:

$$F_R = F_2 - F_1$$
$$F_R = \rho g(h + x)A - h\rho g A$$
$$\therefore F_R = \rho g x A = upthrust$$

But the $height\ of\ the\ block\ (x)$ times the $cross-sectional\ area, A$ will give us the volume V, so:

$$F_R = Upthrust = \rho g V$$

And density (ρ) times the volume (V) gives us the mass therefore:

$$F_R = Upthrust = mg = Weight$$

So again Archimedes' principle holds true in stating upthrust exerted on an object is equal to the weight of the fluid it displaces.

Sink, Float or Rise

- Weight > Upthrust => SINK
- Weight = Upthrust => Float
- Weight < Upthrust => RISE

UPTHRUST

Worked example:

A ship on River Thames is 70m long and 20m wide. What depth of the hull will be under water if it and its cargo have a combined mass of 1.2×10^6 kg? (Assume that the density of water in the River Thames = $1000\ kgm^{-3}$).

To float:

Upthrust = weight

Weight = $mg = 1.2 \times 10^6 \times 9.81 = 1.18 \times 10^7\ N$

$$\therefore upthrust = 1.18 \times 10^7 N$$

The upthrust is equal to the weight of the volume of water displaced by the hull:

$$Upthrust = \rho V g$$

Where volume, $V = length\ of\ hull, l \times width\ of\ hull, w \times depth\ of\ hull\ under\ water, d$

So:

$$upthrust = 1000 \times 70 \times 20 \times d \times 9.81$$
$$1.18 \times 10^7 N = 1.37 \times 10^7 \times d$$
$$d = \frac{1.18 \times 10^7}{1.37 \times 10^7}$$
$$\therefore d = 0.86\ m$$

The hull will be 0.86 m underwater.

Please see '4.1.2 Density and Upthrust Worked Examples' pack for exam style questions.

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