



# AS Level Physics

Chapter 5 – Mechanics

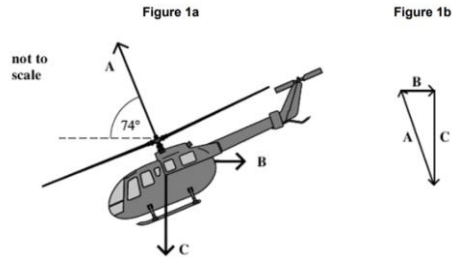
5.6.2 Equilibrium

Worked Examples

## TRIANGLE OF FORCES

### Exam Style Question 1:

a) The helicopter shown in Fig 1a is moving horizontally through still air. The lift force from the helicopter's blades is labelled A.



- Name the two forces B and C that also act on the helicopter.
- The force vectors are also shown arranged as a triangle in Figure 1b.

State and explain how Fig 1b shows that the helicopter is moving at a constant velocity.

c) The lift force, A, is 9.5 kN and acts at an angle of  $74^\circ$  to the horizontal.

Calculate the weight of the helicopter.

## TRIANGLE OF FORCES

### Exam Style Question 1:

**Answer:**

ai) Name the two forces B and C:

B: Drag/air resistance

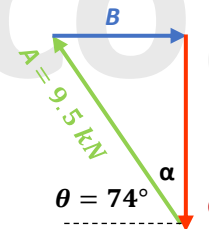
C: Weight

aii) State and explain how Fig 1b shows that the helicopter is moving at a constant velocity.

Because Fig 1b forms a closed triangle this lets us know that the forces are in equilibrium and the resultant force is zero. This means the helicopter is moving at a constant velocity.

c) Calculate the weight of the helicopter.

Step 1: Draw a diagram; mark the relevant angles.



Step 2: Find  $\alpha$ :

$$\alpha = 90^\circ - 74^\circ = 16^\circ$$

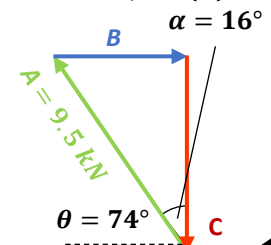
Step 3: Use trigonometry to calculate the weight of the helicopter (C):

$$\cos 16 = \frac{C}{A} = \frac{C}{9500}$$

$$9500 \cos 16 = C$$

$$\therefore C = 9131.986111 = 9131 \text{ N}$$

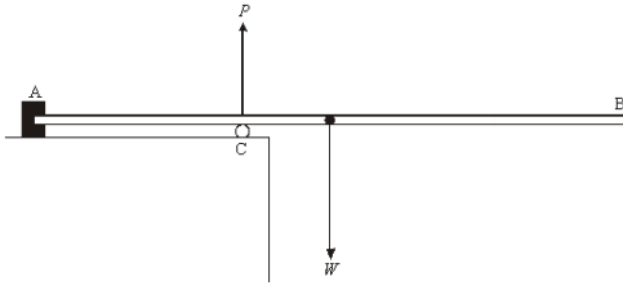
Therefore, the weight of the helicopter is 9131 N.



## MOMENT OF FORCE

### Exam Style Question 2:

- a) Define the moment of a force.
- b) The diagram shows a uniform diving board of weight,  $W$ , that is fixed at  $A$ . The diving board is supported by a cylinder at  $C$ , that exerts an upward force,  $P$ , on the board.



- i) By considering moments about A, explain why the force  $P$  must be greater than the weight of the board,  $W$ .
- ii) State and explain what would be the effect on the force  $P$  of a girl walking along the board from A to B.

## MOMENT OF FORCE

### Exam Style Question 2:

- a) Define the moment of a force.

Product of the force and the perpendicular distance from the fixed point or pivot.

- bi) By considering moments about A, explain why the force  $P$  must be greater than the weight of the board,  $W$ .

$$M = F \times x$$
$$\therefore F = \frac{M}{x}$$

Since  $W$  is at a greater distance from A, then  $W$  must be less than  $P$  if moments are to be equal. This is because  $x$  is inversely proportional to the force so as  $x$  is increased force is decreased.

- bii) State and explain what would be the effect on the force  $P$  of a girl walking along the board from A to B.

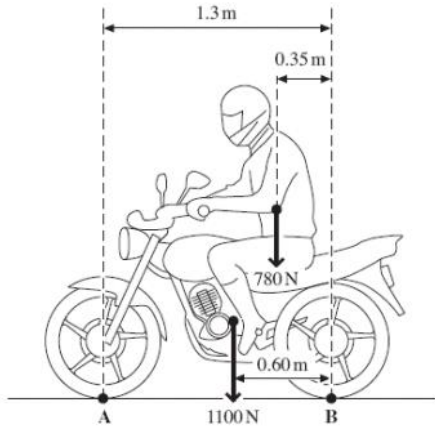
$P$  must increase since the moment acting on the board increases due to the girl's weight as she moves from A to B. Therefore,  $P$  is minimum when the girl is at A and maximum when the girl is at B.



## THE PRINCIPLE OF MOMENTS

### Exam Style Question 3:

The figure below shows a motorcycle and rider. The motorcycle is in contact with the road at A and B.



The motorcycle has a weight of 1100 N and the rider's weight is 780 N.

- State the principle of Moments.
- Calculate the moment of the rider's weight about B. Give an appropriate unit.
- By taking the moments about B, calculate the vertical force that the road exerts on the front tyre at A.
- Calculate the vertical force that the road exerts on the rear tyre at B.



## THE PRINCIPLE OF MOMENTS

### Exam Style Question 3:

#### Answer

a) State the principle of moments.

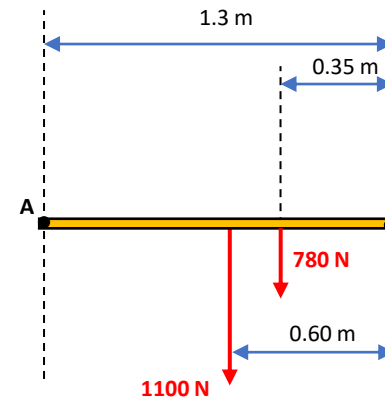
For an object/system to be in equilibrium the sum of clockwise moments about a point is equal to the sum of anticlockwise moments at that point.

b) Calculate the moment of the rider's weight about B. Give an appropriate unit.

Before we start to do the remainder of the questions remember the below rules:

- Use the principle of moments
- Set up a coordinate system where anticlockwise moment is positive
- In equilibrium  $\sum \text{anticlockwise moments} - \sum \text{clockwise moments} = 0$
- Simplify into a simple beam to make the question easier to answer

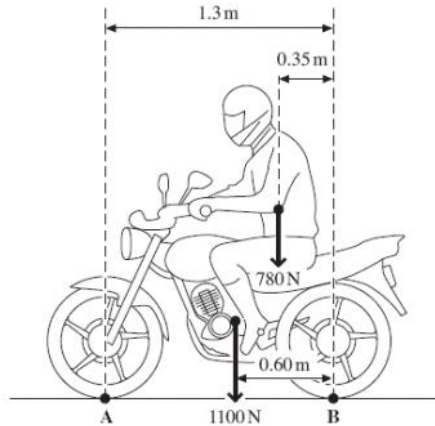
So, let's simplify this complex system into a simple beam as shown below:



## THE PRINCIPLE OF MOMENTS

### Exam Style Question 3:

The figure below shows a motorcycle and rider. The motorcycle is in contact with the road at A and B.



The motorcycle has a weight of 1100 N and the rider's weight is 780 N.

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- By taking the moments about B, calculate the vertical force that the road exerts on the front tyre at A.
- Calculate the vertical force that the road exerts on the rear tyre at B.



## THE PRINCIPLE OF MOMENTS

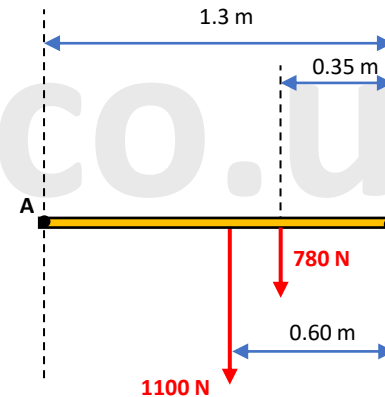
### Exam Style Question 3:

#### Answer

b) Calculate the moment of the rider's weight about B. Give an appropriate unit. (Continued)

- Use the principle of moments
- Set up a coordinate system where anticlockwise moment is positive
- In equilibrium  $\sum \text{anticlockwise moments} - \sum \text{clockwise moments} = 0$
- Simplify into a simple beam to make the question easier to answer

Now using the beam diagram below we can calculate the moment of the rider's weight about B.



This can be done using:

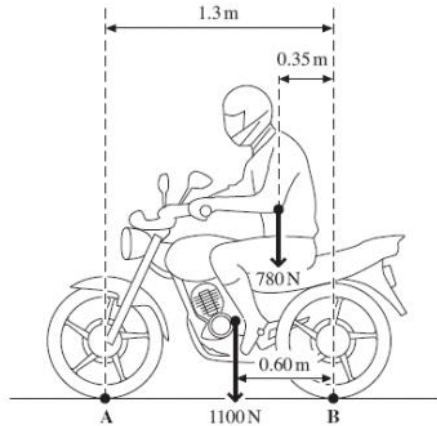
$$M = F \times x$$
$$\curvearrowright M = 780 \text{ N} \times 0.35 \text{ m}$$
$$M = 273 \text{ Nm}$$

Positive because rotation of beam is anti-clockwise

## THE PRINCIPLE OF MOMENTS

### Exam Style Question 3:

The figure below shows a motorcycle and rider. The motorcycle is in contact with the road at A and B.



The motorcycle has a weight of 1100 N and the rider's weight is 780 N.

- State the principle of Moments.
- Calculate the moment of the rider's weight about B. Give an appropriate unit.
- By taking the moments about B, calculate the vertical force that the road exerts on the front tyre at A.
- Calculate the vertical force that the road exerts on the rear tyre at B.

## THE PRINCIPLE OF MOMENTS

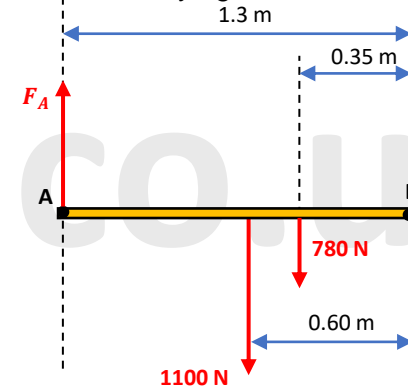
### Exam Style Question 3:

**Answer:**

c) By taking the moments about B, calculate the vertical force that the road exerts on the front tyre at A.

- Use the principle of moments
- Set up a coordinate system where anticlockwise moment is positive
- In equilibrium  $\sum \text{anticlockwise moments} - \sum \text{clockwise moments} = 0$
- Simplify into a simple beam to make the question easier to answer

Step 1: Draw the force we are trying to find on the diagram below:



Step 2: Use  $\sum \text{anticlockwise moments} - \sum \text{clockwise moments} = 0$  about point B:

Sign convention: anticlockwise rotation is positive

$$\curvearrowleft (+) -M_{F_A} + M_{1100N} + M_{780N} = 0$$

$M_{F_A}$  = moment of force  $F_A$   
 $M_{1100N}$   
 = moment of 1100N force  
 $M_{780N}$   
 = moment of 780N force

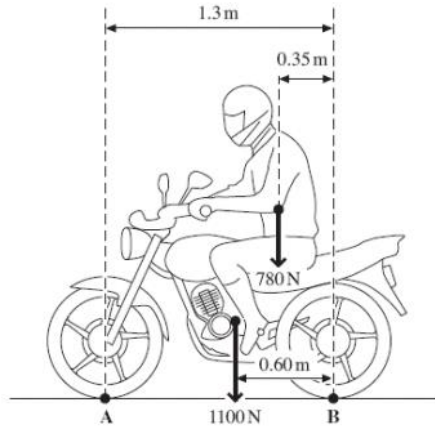
$$\begin{aligned} -(F_A)(1.3) + (1100 \text{ N})(0.60\text{m}) + (780 \text{ N})(0.35 \text{ m}) &= 0 \\ -1.3 F_A + 660 \text{ Nm} + 273 \text{ Nm} &= 0 \\ -1.3 F_A + 933 &= 0 \\ -933 & \\ F_A &= \frac{-933}{-1.3} = 717.7 \text{ N} \end{aligned}$$

Negative sign because rotation of beam is clockwise

## THE PRINCIPLE OF MOMENTS

### Exam Style Question 3:

The figure below shows a motorcycle and rider. The motorcycle is in contact with the road at A and B.



The motorcycle has a weight of 1100 N and the rider's weight is 780 N.

- State the principle of Moments.
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- By taking the moments about B, calculate the vertical force that the road exerts on the front tyre at A.
- Calculate the vertical force that the road exerts on the rear tyre at B.

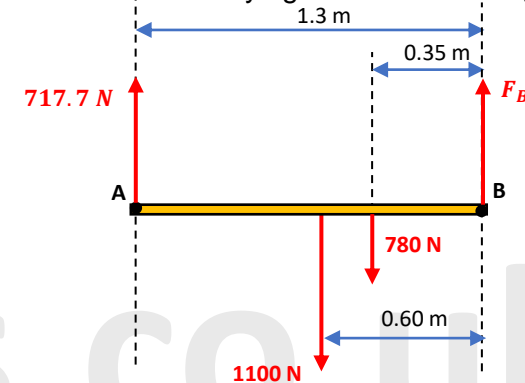
## THE PRINCIPLE OF MOMENTS

### Exam Style Question 3:

#### Answer

d) Calculate the vertical force that the road exerts on the rear tyre at B.

Step 1: Draw the force we are trying to find on the diagram below:



Step 2: Since this system is in equilibrium means the sum of all forces or the resultant force is equal to zero.

$$\sum F_{\text{resultant}} = 0$$

As we know force is a vector quantity, we have to set a coordinate system. So, let's assume all the forces acting downwards are positive. This gives us:

Sign convention:  
downwards force is positive

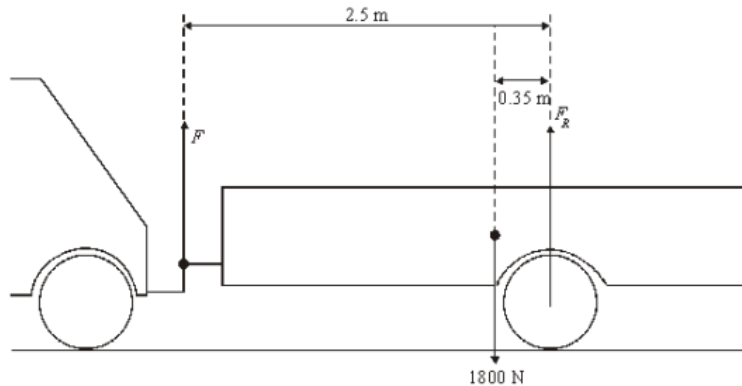
$$+ \downarrow -717.7 + 1100 + 780 - F_B = 0$$
$$1162.3N - F_B = 0$$
$$\therefore F_B = 1162.3 = 1162 N$$

Therefore, the vertical force that the road exerts on the rear tyre at B is 1162 N.

## THE PRINCIPLE OF MOMENTS

### Exam Style Question 4:

- a) The diagram shows a trailer attached to the towbar of a stationary car. The weight of the trailer is  $1800\text{ N}$  and is shown acting through its centre of gravity.  $F$  is the force exerted by the towbar on the trailer.  $F_R$  is the total normal reaction force experienced by the trailer. When stationary all forces acting on the trailer are vertical.



- i) Calculate the force,  $F$ , exerted by the towbar on the trailer.  
ii) Calculate  $F_R$ .
- b) The car starts to move forwards. State and explain what happens to the magnitude and direction of force,  $F$ .

## THE PRINCIPLE OF MOMENTS

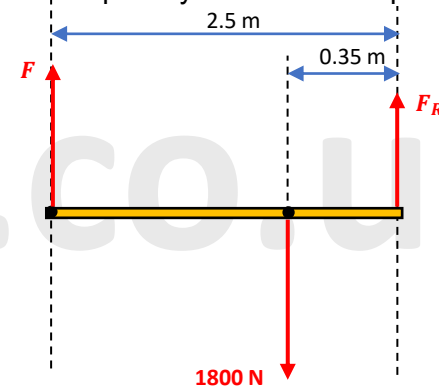
### Exam Style Question 4:

#### Answer

a) State the principle of moments.

- Use the principle of moments
- Set up a coordinate system where anticlockwise moment is positive
- In equilibrium  $\sum \text{anticlockwise moments} - \sum \text{clockwise moments} = 0$
- Simplify into a simple beam to make the question easier to answer

So, let's simplify this complex system into a simple beam like the one below:



As we are trying to find out  $F$  take moments from  $F_R$ .

In equilibrium  $\sum \text{anticlockwise moments} - \sum \text{clockwise moments} = 0$

Therefore:

$$\begin{aligned} \curvearrowright M_{1800} - M_F &= 0 \\ \curvearrowright (1800\text{ N})(0.35\text{ m}) - (F)(2.5\text{ m}) &= 0 \\ 630 - 2.5F &= 0 \\ 630 &= 2.5F \\ \therefore F &= \frac{630}{2.5} = 252\text{ N} \end{aligned}$$

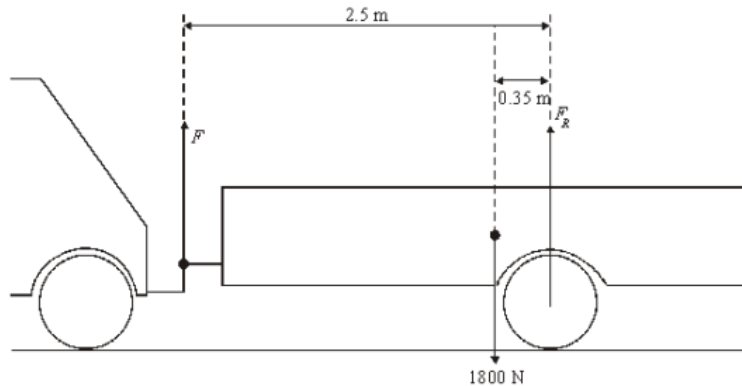




## THE PRINCIPLE OF MOMENTS

### Exam Style Question 4:

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- Calculate the force,  $F$ , exerted by the towbar on the trailer.
  - Calculate  $F_R$ .
- b) The car starts to move forwards. State and explain what happens to the magnitude and direction of force,  $F$ .

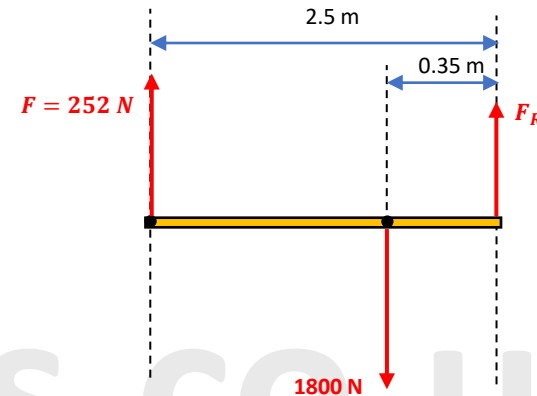


## THE PRINCIPLE OF MOMENTS

### Exam Style Question 4:

#### Answer

(iii) Calculate  $F_R$



Since the car is stationary this system is in equilibrium which means the sum of all forces (resultant force) is equal to zero.

$$\sum F_{\text{resultant}} = 0$$

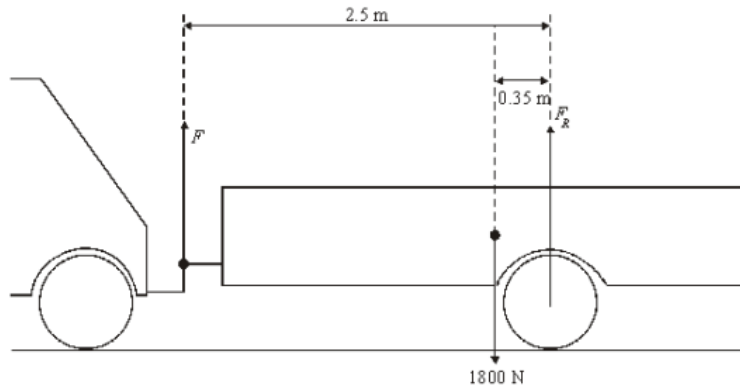
Force is a vector quantity so we have to set a coordinate system. Lets assume all the forces acting downwards are positive. This gives us:

$$\begin{aligned} + \downarrow -252\text{ N} + 1800\text{ N} - F_R &= 0 \\ 1548 - F_R &= 0 \\ \therefore F_R &= 1548\text{ N} \end{aligned}$$

## THE PRINCIPLE OF MOMENTS

### Exam Style Question 4:

- a) The diagram shows a trailer attached to the towbar of a stationary car. The weight of the trailer is  $1800\text{ N}$  and is shown acting through its centre of gravity.  $F$  is the force exerted by the towbar on the trailer.  $F_R$  is the total normal reaction force experienced by the trailer. When stationary all forces acting on the trailer are vertical.



- i) Calculate the force,  $F$ , exerted by the towbar on the trailer.
- ii) Calculate  $F_R$ .
- b) The car starts to move forwards. State and explain what happens to the magnitude and direction of force,  $F$ .

## THE PRINCIPLE OF MOMENTS

### Exam Style Question 4:

#### Answer

- b) **The car starts to move forwards. State and explain what happens to the magnitude and direction of force,  $F$ .**

As the car starts to move forward, the force,  $F$  now has a horizontal component as well as a vertical component. So the Force,  $F$  increases in magnitude and acts at an angle sloping towards the car.



## COUPLES

### Exam Style Question 5:

a) Define the following terms:

- i) Couple
- ii) Torque of a couple

b) Fig 4.1 shows a satellite in space moving from left to right.

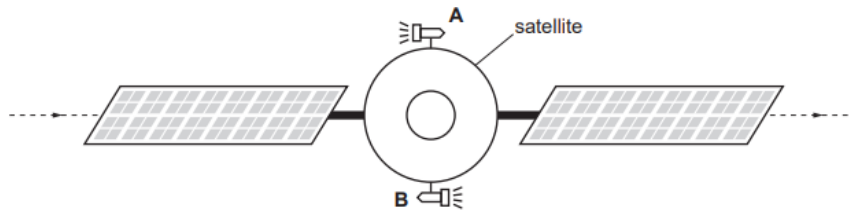


Fig. 4.1

The satellite has two small rockets A and B mounted at opposite ends of a diameter. When fired, each rocket motor provides the same constant force, but in opposite directions.

Describe the change in the motion of the satellite when

- i) Both rocket motors are fired
- ii) Only rocket motor A is fired.



## COUPLES

### Exam Style Question 5:

Answer:

ai) Define a couple:

**Two equal forces** which act **parallel** to each other but in **opposite directions**.

aii) Define a torque of a couple:

Torque = one of the forces  $\times$  perpendicular distance between the forces.

bi) Describe the change in the motion of the satellite when both rocket motors are fired.

The satellite will rotate clockwise and continue to travel from left to right. The rotational speed increases with time.

bii) Describe the change in the motion of the satellite when only rocket motor A is fired.

It will accelerate to the right and the satellite will turn.

## COUPLES

### Exam Style Question 6:

a) Fig. 6.1 shows two equal but opposite forces acting on an object.

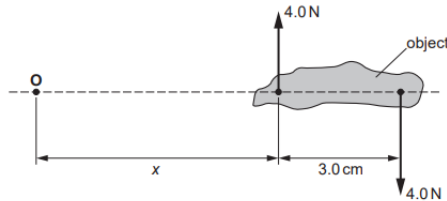


Fig. 6.1

The point O is at a distance  $x$  from the nearer of the two forces.

- i) The separation between the two parallel forces is 3.0 cm. Determine the torque of the couple exerted on the object.
  - ii) Calculate the total moment of the forces about the point O and state the significance of this value.
- b) State two conditionals necessary for an object to be in equilibrium.
- c) Fig 6.2 shows a concrete paving slab in equilibrium.

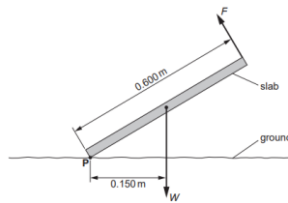


Fig. 6.2

Two forces acting on the slab are shown. The weight of the slab is  $W$ . The force  $F$  is applied at right angles to the end of the slab. By taking moments about  $P$ , determine the size of the force  $F$ .

## COUPLES

### Exam Style Question 6:

**Answer:**

**ai) Determine the torque of the couple exerted on the object.**

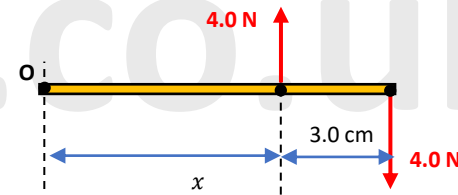
Use the formula:  $T = F \times d$

$$\text{torque} = 4.0 \text{ N} \times 0.03 \text{ m}$$

$$\text{torque} = 0.12 \text{ Nm}$$

**aii) Calculate the total moment of the forces about the point O and state the significance of this value.**

- Use the principle of moments
- Set up a coordinate system where anticlockwise moment is positive
- In equilibrium  $\sum \text{anticlockwise moments} - \sum \text{clockwise moments} = 0$
- Simplify into a simple beam to make the question easier to answer



As we are unsure if this object is in equilibrium we will use "total moment" for now.

Let's assume this system is in equilibrium therefore

$$\sum \text{anticlockwise moments} - \sum \text{clockwise moments} = \text{total moment}$$

$$\curvearrowright M_{4.0 \text{ N}} - M_{4.0 \text{ N}} = \text{total moment}$$

$$\curvearrowright (4.0 \text{ N})(x) - (4.0)(x + 0.03 \text{ m}) = \text{total moment}$$

$$4x - 4x - 0.12 = \text{total moment}$$

$$\therefore \text{total moment} = -0.12 \text{ Nm}$$

The total moment is the same as the torque of the couple calculated in (a)(i).

## COUPLES

### Exam Style Question 6:

a) Fig. 6.1 shows two equal but opposite forces acting on an object.

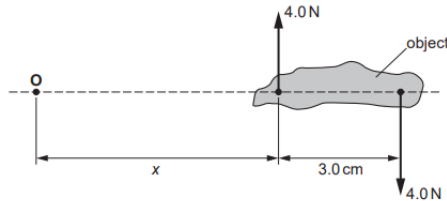


Fig. 6.1

The point O is at a distance  $x$  from the nearer of the two forces.

- The separation between the two parallel forces is 3.0 cm. Determine the torque of the couple exerted on the object.
  - Calculate the total moment of the forces about the point O and state the significance of this value.
- b) State two conditionals necessary for an object to be in equilibrium.
- c) Fig 6.2 shows a concrete paving slab with a mass of 45 kg in equilibrium.

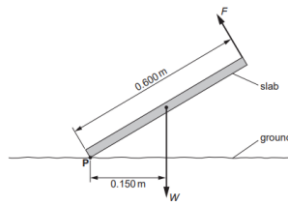


Fig. 6.2

Two forces acting on the slab are shown. The weight of the slab is  $W$ . The force  $F$  is applied at right angles to the end of the slab. By taking moments about  $P$ , determine the size of the force  $F$ .

## COUPLES

### Exam Style Question 6:

Answer:

b) State two conditions necessary for an object to be in equilibrium.

Net (resultant) force = 0.

Clockwise moments = anticlockwise moments.

c) Determine the size of the force,  $F$ .

- Use the principle of moments
- Set up a coordinate system where anticlockwise moment is positive
- In equilibrium  $\sum \text{anticlockwise moments} - \sum \text{clockwise moments} = 0$
- Simplify into a simple beam to make the question easier to answer

Step 1: Calculate  $W$ :

$$W = m \times g$$
$$W = 45 \text{ kg} \times 9.81 \text{ ms}^{-2}$$
$$W = 441.45 \text{ N}$$

Step 2: Take moments about P.

As the slab is in equilibrium therefore:

$$\sum \text{anticlockwise moments} - \sum \text{clockwise moments} = 0$$

$$\curvearrowright M_F - M_W = 0$$

$$(F)(0.600 \text{ m}) - (W)(0.150 \text{ m}) = 0$$

$$(F)(0.600 \text{ m}) - (441.45 \text{ N})(0.150 \text{ m}) = 0$$

$$(F)(0.600 \text{ m}) - 66.2175 = 0$$

$$F = \frac{66.2175}{0.600}$$

$$\therefore F = 110.3625 \text{ N m}$$

$$F = 110 \text{ N}$$



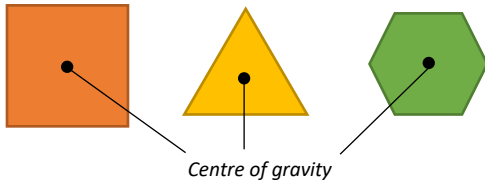
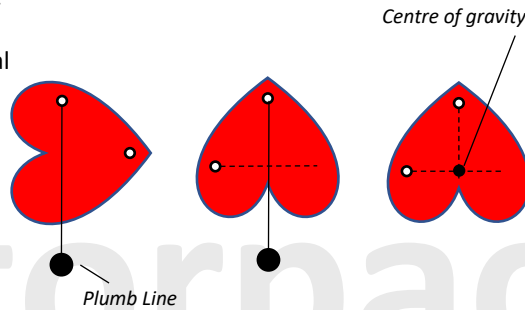
## CENTRE OF GRAVITY

The centre of gravity (or the centre of mass) of an object is the single point at which we can consider its entire weight to act through. In other words a single point where all the mass is concentrated.

You can find the centre of gravity by using symmetry for regular shapes or experimentally for regular and irregular shapes.

### Finding the centre of gravity for an irregular object

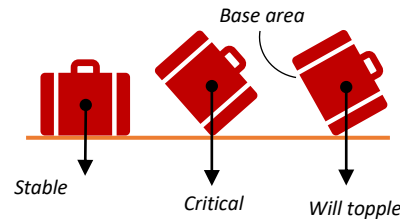
- 1) Freely suspend an object from one corner.
- 2) Then using a plumb, draw a vertical line downwards from the point of suspension.
- 3) Now pick another corner to suspend the object.
- 4) Draw another vertical line downwards.
- 5) Where the two lines intersect is where the centre of gravity for the object is located.



For a regular object you can just use symmetry. The centre of gravity of any regular shape is at its centre.

### Stability

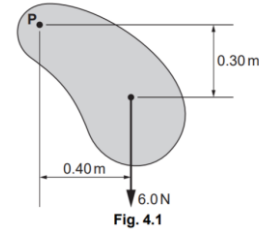
- 1) Stable objects have low centre of gravity and a wide base area.
- 2) Objects will topple over if the line of action from the centre of gravity falls outside the objects base area:



## CENTRE OF GRAVITY

### **Exam Style Question 7:**

Fig 4.1 shows an irregular shaped metal plate of constant thickness that can swing freely about point P.



- a) State what is meant by the centre of gravity of an object.
- b) Describe an experiment to determine the centre of gravity of the metal plate shown in Fig. 4.1.

### **Answer:**

- a) A point where the entire weight of the object appears to act.
- b) Step 1: Suspend the metal plate from a point and then mark a vertical line on the plate. A plumb line or a pendulum can be used to find the vertical line.  
Step 2: Hang the plate from another point and draw another vertical line.  
Step 3: Where the lines intersect gives you the position of the centre of gravity.

Please see **'5.6.1 Equilibriums notes'** pack for  
revision notes.

For more revision notes, tutorials, worked  
examples and more help visit  
[www.tutorpacks.co.uk](http://www.tutorpacks.co.uk).

