AS Level Physics

Chapter 5 – Mechanics

5.4.1 Dynamics

Notes

MASS vs WEIGHT

Mass:

- Mass is a property of any object. It is the amount of matter an object has.
- Mass is measured in kilograms (kg).
- The greater the mass of an object, the greater its resistance to any changes in its velocity (this is known as inertia).
- The mass of an object doesn't change even if the gravitational field strength does. This just means the mass of an object is the same on the Earth and the Moon.

Weight:

- The Earths gravitational pull is what causes us to have weight.
- Weight is a force that always acts perpendicularly downwards.
- Weight is measured in Newtons (like all forces).
- Weight is dependent on two factors: the mass (m) of an object and the gravitational field strength, (g).
- The weight of an object changes when the gravitational field strength changes. This means that the weight of an object will differ on the Moon and on the Earth.
- Weight is calculated using the formula below:

$$weight(N) = mass(kg) \times gravitational\ field\ strength\ (ms^{-2}\ or\ Nkg^{-1})$$
$$W = mg$$

- On Earth the gravitational field strength is $9.81\ ms^{-2}$ or $9.81\ Nkg^{-1}$.

ms^{-2} is the same as Nkg^{-1}.
Why?
$$Nkg^{-1} = \frac{N}{kg^{-1}} = \frac{kg\ ms^{-2}}{kg} = \mathrm{ms}^{-2}$$

MASS vs WEIGHT

Example 1:

What is the weight of a 80kg person on Earth and the Moon?

On Earth the gravitational field strength is $9.81\ ms^{-2}$ therefore a person on Earth will have a:

$$Mass = 80\ kg$$
$$Weight = 80\ kg \times 9.81\ ms^{-2} = 784.8\ N$$

On the Moon the gravitational field strength is $1.6\ ms^{-2}$ therefore a person on the Moon will have a:

$$Mass = 80\ kg$$
$$Weight = 80\ kg \times 1.6\ ms^{-2} = 128\ N$$

Can you see how the mass of the person doesn't change on the Earth or the Moon however the weight of the person does. The weight of the person changes because the Earth and the Moon have a different gravitational field strength.

FREE BODY DIAGRAMS (FBD)

- A FBD shows the magnitude and direction of all the forces acting on an object in a given situation.
- In other words it is a diagram of a single body with all the forces that are acting on that body.
- Remember that forces are vector quantities so the magnitude and the direction matter. To represent these forces you use arrows. The size of the arrow represents the magnitude and whatever direction the arrow is pointing towards represents the direction. The bigger the arrow the bigger the magnitude.
- If all the forces, acting on an object, are balanced then the object isn't accelerating and the object is said to be in equilibrium.

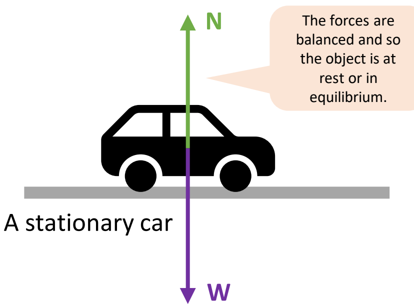
Before starting to draw FBDs you need to know some **important terms**:

1) Weight (W) – Force due to gravity. Pulls objects towards the centre of the Earth. Weight always acts directly downwards from the centre of gravity of an object.
2) Normal (contact) force (N) – The force between two objects touching. This force acts perpendicularly upwards on a object in contact with a surface. This is the counter force to weight.
3) Driving, Engine or Forward force (D) – Causes an object to move forward.
4) Friction Force (F) – A resistive force which acts between two surfaces that slide, across each other. They push parallel to the contact surface and act in the opposite direction of sliding.
5) Air resistance (A) – Type of frictional force between air and another material.
6) Tension (T) – A pulling force caused by a string, rope, cable, wire, etc… The force pulls along the direction of the rope on the object.
7) Buoyancy (B) – A force of the water pushing up (upthrust). Causes objects or materials to float.

FREE BODY DIAGRAMS (FBD)

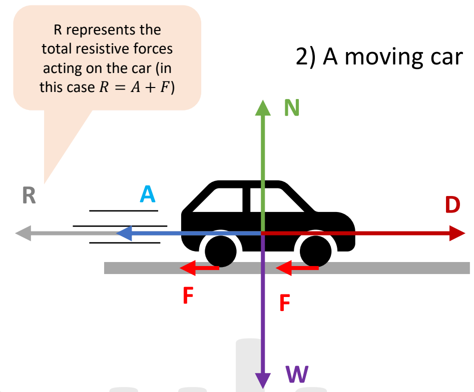
Examples:

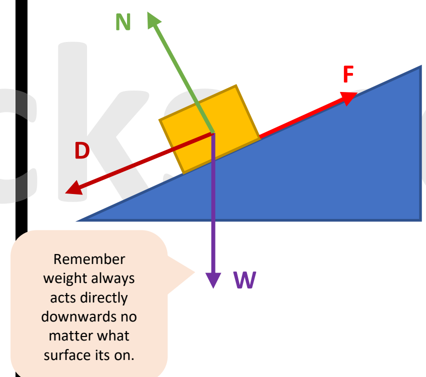
Below are a few common examples of FBD to help you get started:



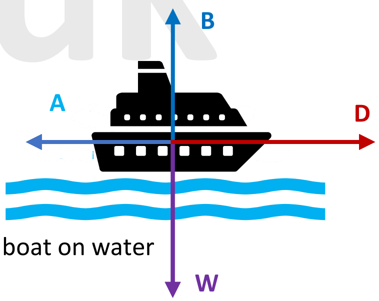
1) A stationary car

2) A moving car

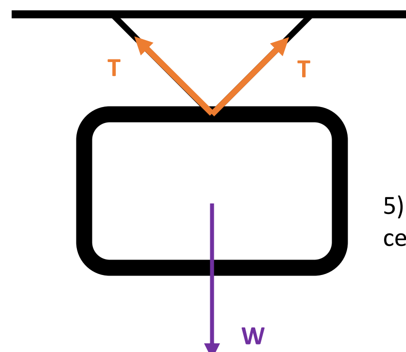




3) A box on an inclined plane.



4) A boat on water



5) A photo frame on a ceiling

NET FORCE

What is a force?

- A force is any interaction that causes stationary objects to move, speed up or slow down.
- Forces are also present when you pull or push objects.
- When all the forces acting on an object are equal the object is moving at a constant velocity, or the object is stationary.
- When an object is stationary or moving with a constant velocity means that the object is in equilibrium, but more on that later.

How to calculate force?

$$Resultant\ (or\ net)\ Force = mass \times acceleration$$

or

$$F = ma$$

Where:

- $F = Resultant\ force\ measured\ in\ \boldsymbol{Newtons}\ (\boldsymbol{N})$
- $m = mass\ measured\ in\ kilograms\ (kg)$
- $a = acceleration\ measured\ in\ metres\ per\ second\ squared\ (ms^{-2})$

This means that the resultant force is directly proportional to the mass and acceleration of an object. This also means that the acceleration is inversely proportional to the mass. So if you double the mass you will be halving the acceleration.

NET FORCE

THE NEWTON (N)

Force is measured in Newtons so using the formula F = ma we can define the Newton as:

$$1\ N = 1\ kg \times 1\ ms^{-2}$$

1 Newton is the force that gives a mass of 1kg an acceleration of $1\ ms^{-2}$.

Knowing this information we can derive the Newton in its SI units forms:

$$N = kg\ ms^{-2}$$

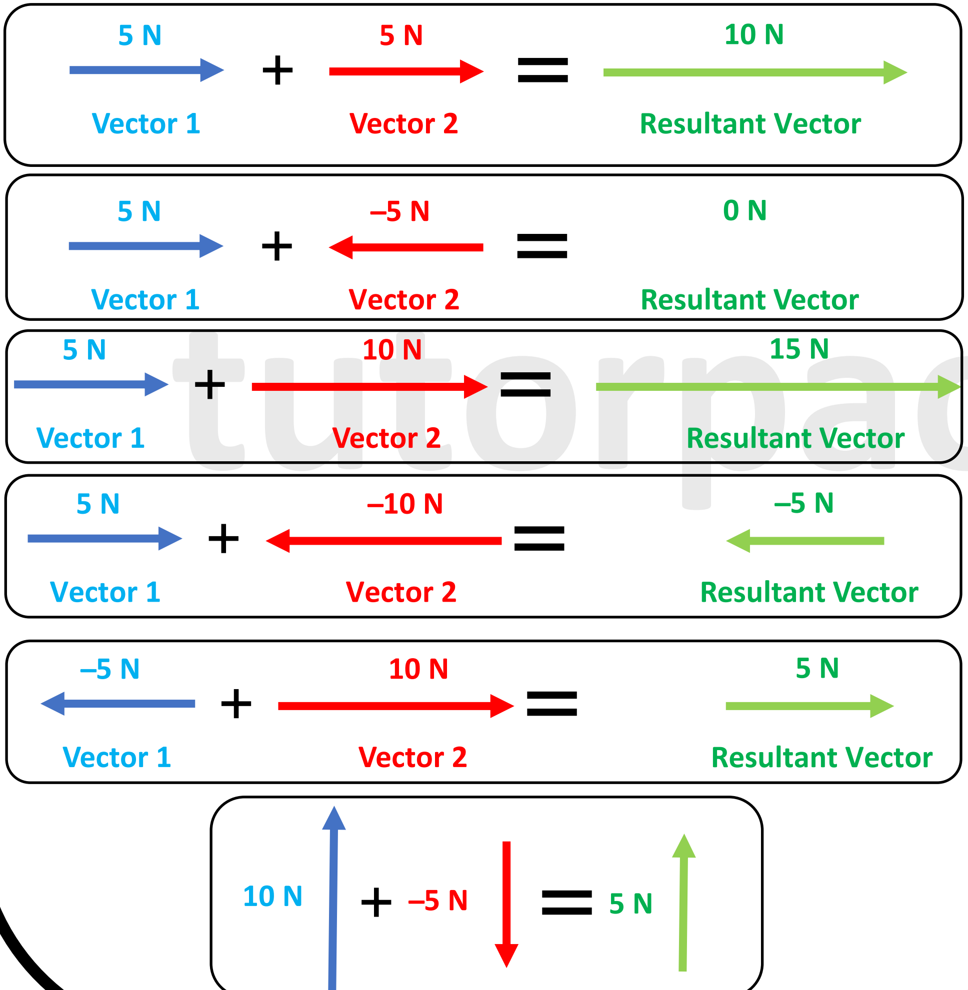
Note:

In simple terms, the resultant force is the vector sum of all the forces acting on an object.

1. The force is a vector quantity.
2. The resultant force of a number of forces is that single force which has the same effect, in both magnitude and direction, as the sum of the individual forces.
3. The acceleration is always in the same direction as the resultant force. E.g. if the resultant force is to the right the acceleration of the object is also to the right, and vice versa.

One Dimensional

Remember that force is a **vector quantity** where magnitude and direction can be both represented by arrows. Please see pack "**Scalar and Vector quantity**" for more detail on resolving vectors.



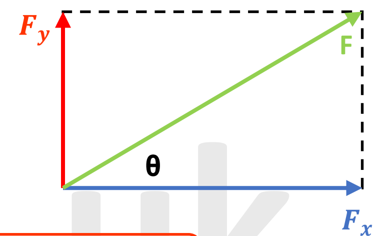
RESOLVING FORCES

Two Dimensional

To analyse a force, it is essential to "break-up" or resolve a force into its horizontal and vertical components.

Consider a Force, F at an angle θ to the x-axis. The Force F can be resolved into its horizontal (x) and vertical (y) components using trigonometry:

The diagram opposite shows Force, F which has been resolved into a vertical (F_y) component and a horizontal (F_x) component.



To calculate the components you use the formulas shown below:

Vertical Component: $F_y = F\, sin\theta$

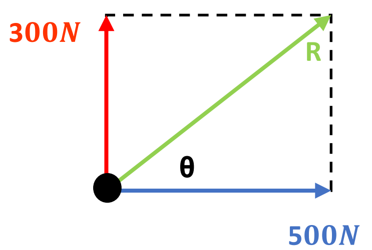
Horizontal Component: $F_x = F\, cos\theta$

RESOLVING FORCES

Example 1:

Two ducks pull a heavy mass. One gives 300 N force North, the other 500 N East. What is the resultant and the direction of the force?

1) Draw the FBD diagram:



2) Calculate the resultant force (R) using Pythagoras:

$$R^2 = 500^2 + 300^2$$
$$R = \sqrt{500^2 + 300^2}$$
$$\mathbf{R = 583.1\ N\ (1\ d.p.)}$$

3) Calculate the angle θ to obtain the direction using trigonometry:

$$tan\theta = \frac{300}{500}$$
$$\theta = \tan^{-1}\left(\frac{300}{500}\right)$$
$$\boldsymbol{\theta = 31^{\circ}}$$

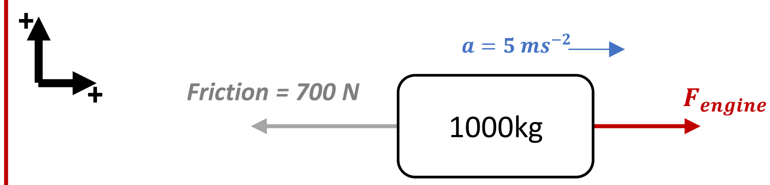
The resultant force is 583N acting at 31 degrees from the horizontal.

RESOLVING FORCES

Example 2:

A 1000kg car accelerates horizontally at $5ms^{-2}$.
The car has a 700N frictional force acting on it.
Calculate the force of the engine acting against the frictional force.

1) Draw the FBD diagram and set up the coordinate system:



2) Calculate the resultant force of the whole system using F=ma:

$$F_{resultant} = mass \times accelecration$$
$$F_{resultant} = 1000\ kg \times 5\ ms^{-2}$$
$$F_{resultant} = 5000N$$

Remember this resultant force acts to the right because the resultant force always acts in the same direction as the acceleration.

3) The next step is to calculate the engine force, F_{engine}:

$$F_{engine} - Friction = F_{resultant}$$
$$F_{engine} - 700N = 5000N$$
$$F_{engine} = 5000 + 700$$
$$F_{engine} = 5700N$$

Remember: the resultant is the vector sum of all the forces acting on an object.

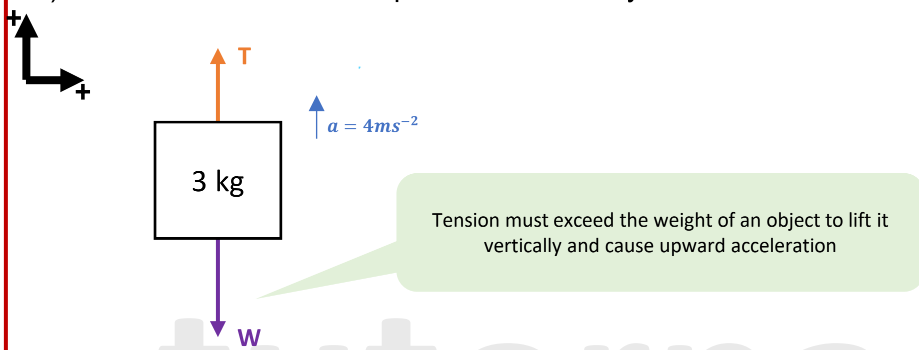
Therefore the engine force acting on the car is 5700N.

RESOLVING FORCES

Example 3:

A 3kg object is pulled vertically upwards by a rope. The mass accelerates at 4 ms^{-2}. Calculate the tension in the rope.

1) Draw the FBD and set up the coordinate system:



2) Calculate the weight force acting on the object using W=mg:

$$Weight = mass \times acceleration\ due\ to\ gravity$$
$$W = m \times g$$
$$W = 3kg \times 9.81ms^{-2}$$
$$W = 29.4N$$

The value of g = 9.81m/s² on Earth.

3) Calculate the resultant force of the whole system using F=ma:

$$F_{resultant} = 3kg \times 4\ ms^{-2}$$
$$F_{resultant} = 12N$$

This resultant force acts upwards because the resultant force always acts in the same direction as the acceleration.

4) Calculate the Tension force, T:

$$Tension - Weight = F_{resultant}$$
$$T - 29.4N = 12\ N$$
$$T = 12\ N + 29.4\ N$$
$$\boldsymbol{T = 41.4N}$$

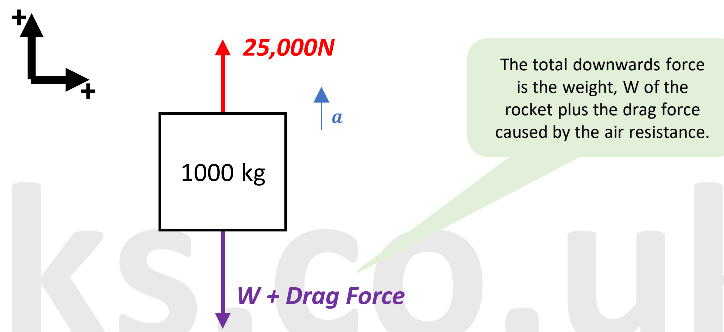
Tension is positive (upward) and weight is negative (downward) due to the coordinate system. Resultant force is positive (upward) in alignment with acceleration

RESOLVING FORCES

Rocket Motion Example 1:

A missile has a mass of 1000 kg and is fired vertically into the air.
Its rockets provide a thrust of 25,000N.
The drag force caused by air resistance is 3,000N.
Calculate the acceleration of the missile.

1) Draw the FBD and set up the coordinate system:



2) Calculate the total downwards force:

$$Weight = 1000\ kg \times 9.81\ ms^{-2}$$
$$Weight = 9810N$$
$$\boldsymbol{Total\ downwards\ force = 9810N + 3000N = 12810N}$$

3) Calculate the total resultant force, $F_{resultant}$:

$$F_{resultant} = \boldsymbol{25,000} - \boldsymbol{12,810}$$
$$F_{resultant} = 12190\ N$$

The resultant force is the vector sum of all the individual forces acting on the object. So using the coordinate system find the resultant force.

4) Now using $F = ma$ you can calculate the acceleration:

$$F_{resultant} = m \times a$$
$$12190\ N = 1000kg \times a$$
$$a = \frac{12190}{1000} = 12.19\ ms^{-2}$$

So the acceleration of the rocket is $\boldsymbol{12.19\ ms^{-2}}$.

RESOLVING FORCES

Rocket Motion Example 2:

A rocket of mass 600kg is launched from Cape Canaveral. The total engine thrust is 9000N.

a) Calculate the acceleration of the rocket.

b) The acceleration of the rocket increases as the rocket gains altitude. Explain why?

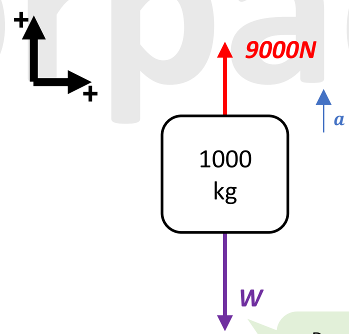
c) The same rocket takes off from the moon where gravity is 1.6 $N\,kg^{-1}$. Calculate the new initial acceleration.

d) On Jupitar, gravity is 26 $N\,kg^{-1}$. Explain whether this rocket will be able to take off or not.

a) Calculate the acceleration of the rocket:

$W = mg$
$W = 600kg \times 9.81\ ms^{-2}$
$\mathbf{W = 5880N}$

Nkg^{-1} is the same as ms^{-2}.



$F_{resultant} = m \times a$
$\mathbf{9000N} - \mathbf{5880N} = 600kg \times a$
$3120N = 600kg \times a$
$a = \frac{3120}{600}$
$\mathbf{a = 5.2\ ms^{-2}}$

Do not include any drag force as no drag force was mentioned.

The acceleration of the rocket is 5.2 ms^{-2}.

RESOLVING FORCES

Rocket Motion Example 2:

b) The acceleration of the rocket increases as the rocket gains altitude. Explain why?

The mass of the rocket decreases, as the fuel on board the rocket is used up, so weight decreases.
Therefore the size of the resultant force increases as weight decreases ($F_{resultant} = 9000N - \downarrow W$).
So using $a = \frac{F_{resultant}}{m}$ as the resultant force increases and the mass decreases, acceleration increases.

Other reasons may include:

- Air resistance decreases at higher altitudes.
- The gravitational field strength is weaker at higher altitudes.

c) Calculate the initial acceleration on the moon.

$$W = mg$$
$$W = 600kg \times 1.6\ Nkg^{-1}$$
$$W = 960N$$

For this question you will need to re-calculate the weight of the rocket because the acceleration due to gravity on the moon is $1.6Nkg^{-1}$ rather then the normal 9.81 Nkg^{-1} like on Earth.

$$F_{resultant} = m \times a$$
$$9000N - 960N = 600kg \times a$$
$$8040N = 600kg \times a$$
$$\mathbf{a = \frac{8040}{600} = 13.4\ ms^{-2}}$$

d) Explain whether this rocket will take off on Jupitar:

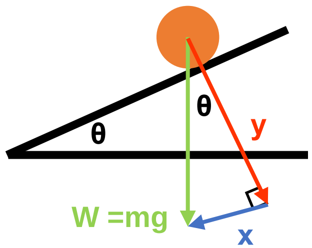
$$W = m \times g$$
$$W = 600 \times 26\ Nkg^{-1}$$
$$W = 15600N$$

The rockets engine thrust is only 9000N whereas the weight of the rocket on Jupitar is 15600N. Therefore as the (downwards) weight force is greater than the (upwards) engine thrust force, the rocket will not take off.

RESOLVING FORCES

Force components on a slope:

For a detailed notes of force components on a slope check out **"2.3 Scalar and Vector Quantity"** pack.

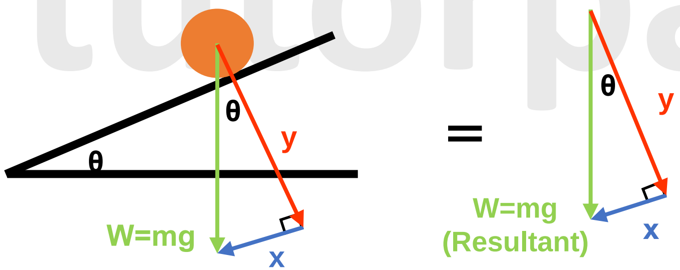


A ball will fall freely towards the Earth due to its weight (W = mg).

The weight of a ball placed on a slope can be split into **two components**.

One component is **PARALLEL** to the slope, the other is **PERPENDICULAR** to the slope.

The **PARALLEL** component makes the ball **run down** the slope.
The **PERPENDICULAR** component **holds** the ball against the slope.



Perpendicular Component (y):

$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{y}{W} = \frac{y}{mg}$$

$$So\ \boldsymbol{y = mg\cos\theta}$$

Parallel Component (x):

$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{x}{W} = \frac{x}{mg}$$

$$So\ \boldsymbol{x = mg\sin\theta}$$

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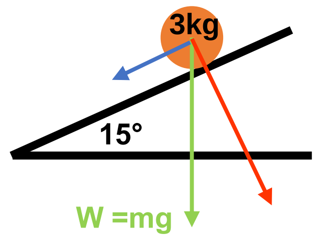
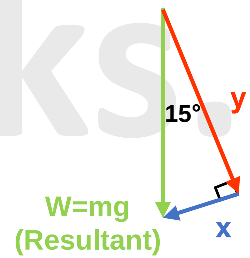
RESOLVING FORCES

Force components on a slope – Example 1:

A 3kg ball sits on a 15° frictionless slope. Calculate the acceleration of the ball.



1) Find the **parallel component** of the weight because this is the component that causes the ball to run down the slope and accelerate:



Parallel Component:

$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{x}{W} = \frac{x}{mg}$$

The value of g = 9.81m/s² on Earth.

$$\sin 15 = \frac{x}{(3kg)(9.81ms^{-2})} = \frac{x}{29.43N}$$

$$So\ \boldsymbol{x = 29.43\sin 15 = 7.62\ N}$$

2) Calculate the force now using $F = ma$:

$$F_{resultant} = ma$$

$$a = \frac{F_{resultant}}{m}$$

$$a = \frac{7.62}{3}$$

$$a = 2.54\ ms^{-2}$$

So the acceleration of the ball is **2.54 ms^{-2}**.

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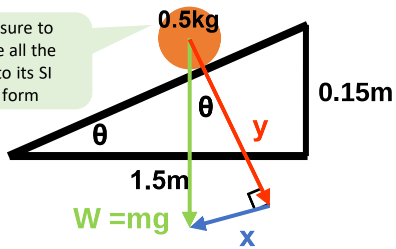
RESOLVING FORCES

Force components on a slope – Example 2:

A 500g tennis ball runs down a runway which is 1.5m long and is raised up by 15cm at one end. The ball's speed remains constant throughout. Calculate the force of friction acting on the slope.

Make sure to change all the units to its SI unit form



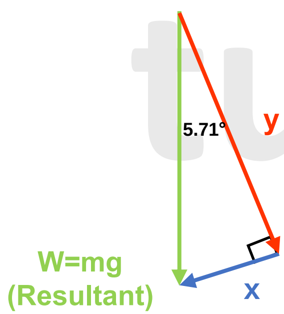
1) Calculate the angle **θ** of the slope:

$$tan\theta = \frac{opposite}{adjacent} = \frac{0.15}{1.5}$$

$$tan\theta = 0.10$$

$$\theta = \tan^{-1}(0.10)$$

$$\boldsymbol{\theta = 5.71° \; (2\,d.p.)}$$



2) Calculate the parallel component:

$$sin\theta = \frac{opposite}{hypotenuse} = \frac{x}{W} = \frac{x}{mg}$$

$$\sin(5.71) = \frac{x}{(0.5kg)(9.81ms^{-2})} = \frac{x}{4.9N}$$

$$So\ \boldsymbol{x = 4.9\,sin(5.71) = 0.488\,N\ (3\,d.p)}$$

3) Calculate the frictional force:

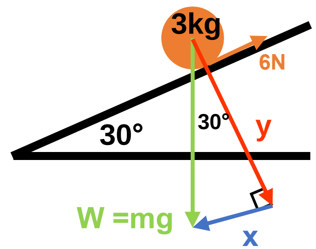
$$constant\ speed = balanced\ force$$
$$Friction\ force = parallel\ component\ (x)$$
$$\boldsymbol{Friction\ force = 0.488\ N}$$

Friction acts in parallel with the slope and as the forces are balanced, the frictional force equals to the parallel component (x).

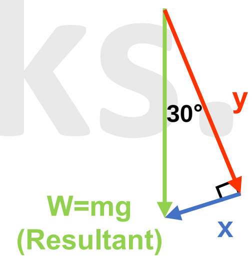
RESOLVING FORCES

Force components on a slope – Example 3:

A 3kg ball slides down a 30° slope. The force of friction acting on the block is 6N. Calculate the acceleration of the ball down the slope.



1) Find the **parallel component** of the weight because this is the component that causes the ball to run down the slope and accelerate:



Parallel Component:

$$sin\theta = \frac{opposite}{hypotenuse} = \frac{x}{W} = \frac{x}{mg}$$

$$\sin(30) = \frac{x}{(3kg)(9.81ms^{-2})} = \frac{x}{29.43\ N}$$

$$So\ \boldsymbol{x = 29.43\,sin(30) = 14.715\ N}$$

2) Calculate the force now using F=ma:

$$F_{resultant} = 14.715 - 6 = 8.715\ N$$
$$F_{resultant} = ma$$
$$a = \frac{F_{resultant}}{m}$$
$$a = \frac{8.715}{3}$$
$$a = 2.905\ ms^{-2}$$

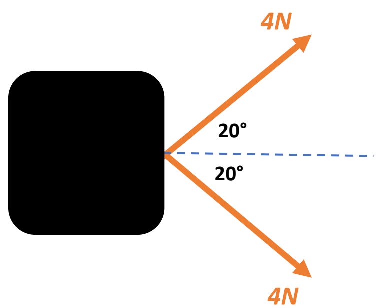
You will need to find the resultant force first by taking away the parallel component from friction and then you can use F=ma.

So the acceleration of the ball is **2.9 ms^{-2}**.

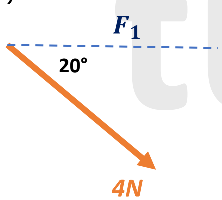
RESOLVING FORCES

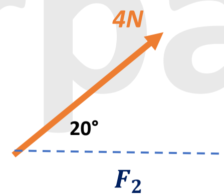
Resultant of Two Forces – Example 1:

Two forces act on an object as shown. Find the resultant of these forces.



1) Resolve each component separately first:





$$cos20 = \frac{F_1}{4}$$

$$F_1 = 4 \times cos20$$

$$\boldsymbol{F_1 = 3.76\ N\ (2\ d.p.)}$$

$$cos20 = \frac{F_2}{4}$$

$$F_2 = 4 \times cos20$$

$$\boldsymbol{F_2 = 3.76\ N\ (2\ d.p.)}$$

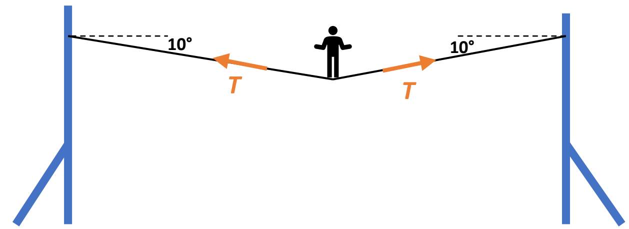
2) Add F_1 and F_2 to find the total resultant force F:

Resultant force is 7.52 N horizontally to the right.

RESOLVING FORCES

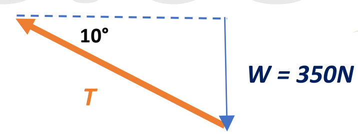
Resultant of Two Forces – Example 2:

An acrobat is stationary at the centre of a tightrope. The acrobat weighs 700N. The angle between the rope and the horizontal is 10° as shown. Calculate the tension T of the rope.



1) Resolve the force first:

Weight of 700N is equally divided between two ropes at the middle, resulting in 350N each.



$$\sin(10) = \frac{350}{T}$$

$$T = \frac{350}{\sin(10)}$$

Equally split weight leads to equally split tension.

$$\boldsymbol{T = 2.02 \times 10^3\ N\ (2\ d.p.)}$$

Elevator Physics (Lift Motion)

Elevator Physics (Lift Motion):

Objects in a lift can accelerate, travel at a constant speed or decelerate.

In a lift, your weight feels heavier than normal when:

- **Accelerating upwards (or downwards)**

In a lift, your weight feels lighter than normal when:

- **Decelerating downwards (or upwards)**

In a lift, your weight feels normal when:

- **Stationary or travelling at constant speed**

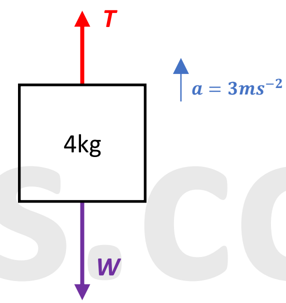
Elevator Physics (Lift Motion)

Lift Motion Questions – Example 1:

A package of mass 4kg is connected to a Newton balance which is attached to the ceiling of a lift.

Calculate the reading on the Newton balance at each stage of the following journey.

a) Accelerates at 3 ms^{-2} upwards



The Newton balance measures the upwards force produced by the tension (T) in the spring.

1) Calculate the weight:

$$\boldsymbol{W = mg}$$
$$W = 4\,kg \times 9.81\,ms^{-2}$$
$$\boldsymbol{W = 39.2\,N}$$

2) Calculate the resultant force for the system:

$$\boldsymbol{F_{resultant} = ma}$$
$$F_{resultant} = 4\,kg \times 3\,ms^{-2}$$
$$\boldsymbol{F_{resultant} = 12\,N}$$

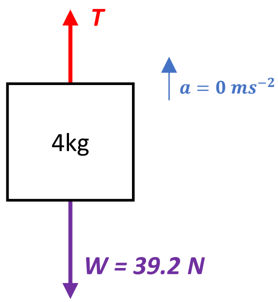
3) As the lift accelerates upwards your weight feels heavier than normal therefore the upwards force will be 12N greater than the downwards force:

$$T = 39.2N + 12N$$
$$\boldsymbol{T = 51.2\,N}$$

Elevator Physics (Lift Motion)

Lift Motion Questions – Example 1:

b) Travels with a constant velocity upwards



At constant velocity:

- **Acceleration is zero**

$$F_{resultant} = ma$$
$$F_{resultant} = 4kg \times 0ms^{-2}$$
$$F_{resultant} = 0\ N$$

This means at constant velocity all the forces are balanced. So:

Tension in rope = weight of package

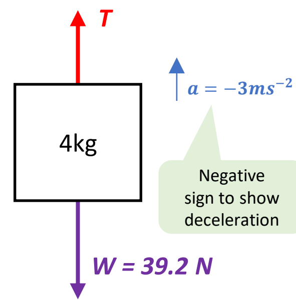
$$Tension = Weight$$

$$\boldsymbol{Tension = 39.2\ N}$$

Elevator Physics (Lift Motion)

Lift Motion Questions – Example 1:

c) Decelerates at $3ms^{-2}$ upwards



Calculate the resultant force for the system:

$$\boldsymbol{F_{resultant} = ma}$$
$$F_{resultant} = 4\ kg \times (-3)\ ms^{-2}$$
$$\boldsymbol{F_{resultant} = -12\ N}$$

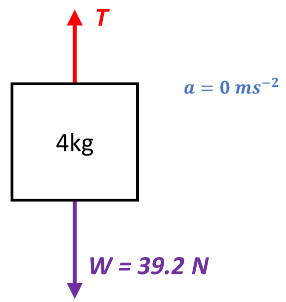
3) As the lift decelerates upwards your weight feels lighter than normal therefore the downwards force will be 12N greater than the upwards force:

$$T = 39.2N - 12N$$
$$\boldsymbol{T = 27.2\ N}$$

Elevator Physics (Lift Motion)

Lift Motion Questions – Example 1:

d) Stopped



When the lift is stationary or stopped **acceleration is zero.**

$$F_{resultant} = ma$$
$$F_{resultant} = 4kg \times 0ms^{-2}$$
$$F_{resultant} = 0\ N$$

This means at all the forces are balanced. So:

Tension in rope = weight of package

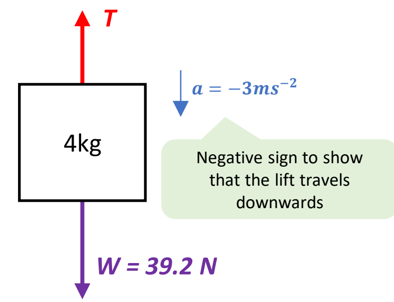
$$Tension = Weight$$

$$\boldsymbol{Tension = 39.2\ N}$$

Elevator Physics (Lift Motion)

Lift Motion Questions – Example 1:

e) Accelerates at $3ms^{-2}$ downwards



Calculate the resultant force for the system:

$$\boldsymbol{F_{resultant} = ma}$$
$$F_{resultant} = 4\ kg \times (-3)\ ms^{-2}$$
$$\boldsymbol{F_{resultant} = -12\ N}$$

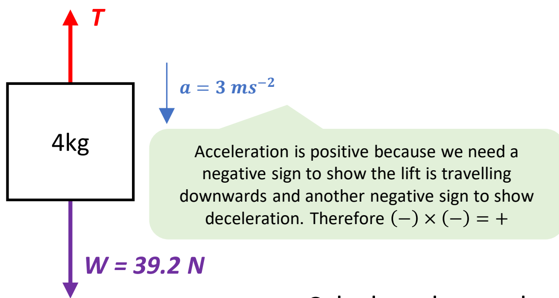
Tension:

$$T = 39.2N - 12N$$
$$\boldsymbol{T = 27.2\ N}$$

Elevator Physics (Lift Motion)

Lift Motion Questions – Example 1:

f) Decelerates downwards at 3 ms^{-2}



Calculate the resultant force for the system:

$$\boldsymbol{F_{resultant} = ma}$$
$$F_{resultant} = 4\ kg \times 3\ ms^{-2}$$
$$\boldsymbol{F_{resultant} = 12\ N}$$

Tension:

$$T = 39.2N + 12N$$
$$\boldsymbol{T = 51.2\ N}$$

Elevator Physics (Lift Motion)

Lift Motion Questions – Example 2:

A person of mass 75 kg enters a lift.

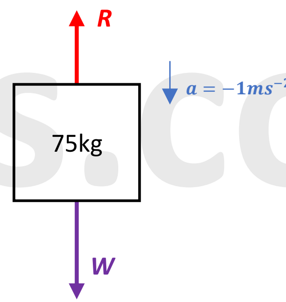
He enters the lift and the lift descends with an acceleration of 1 ms^{-2}.

The lift then descends at a steady speed before coming to rest with a deceleration of 1 ms^{-2}.

a) Calculate the force exerted on the person by the floor when the lift is accelerating.

b) Calculate the force exerted on the person by the floor when the lift is decelerating.

a)



1) Calculate the weight:

$$\boldsymbol{W = mg}$$
$$W = 75\ kg \times 9.81\ ms^{-2}$$
$$\boldsymbol{W = 735\ N}$$

2) Calculate the resultant force for the system:

$$\boldsymbol{F_{resultant} = ma}$$
$$F_{resultant} = 75\ kg \times -1\ ms^{-2}$$
$$\boldsymbol{F_{resultant} = -75\ N}$$

3) Calculate the reaction force (R)

$$R = 735N - 75N$$
$$\boldsymbol{R = 660\ N}$$

Elevator Physics (Lift Motion)

Lift Motion Questions – Example 2:

A person of mass 75 kg enters a lift.

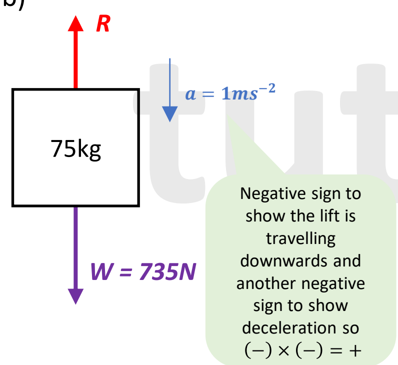
He enters the lift and the lift descends with an acceleration of 1 ms^{-2}.

The lift then descends at a steady speed before coming to rest with a deceleration of 1 ms^{-2}.

a) Calculate the force exerted on the person by the floor when the lift is accelerating.

b) Calculate the force exerted on the person by the floor when the lift is decelerating.

b)



1) Calculate the resultant force for the system:

$$\boldsymbol{F_{resultant} = ma}$$
$$F_{resultant} = 75\ kg \times 1\ ms^{-2}$$
$$\boldsymbol{F_{resultant} = 75\ N}$$

3) Calculate the reaction force (R)

$$R = 735N + 75N$$
$$\boldsymbol{R = 735\ N}$$

Please see the '5.4.2 Dynamics Worked Examples' pack for exam style questions.

Please see the '5.4.2 Dynamics Worked Examples' pack for exam style questions.

For more revision notes, tutorials, worked examples and more help visit www.tutorpacks.co.uk.