A2 Level Physics

Chapter 11 – Gravitational Fields

11.1.2 Gravitation and Planetary Motion

Worked Examples

Gravitational Fields

Exam Style Question 1

(a) Fig. 2.1 shows the Earth in space.

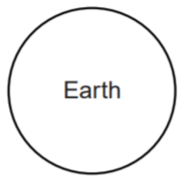


Fig. 2.1

(i) Draw lines on Fig. 2.1 to show the shape and direction of the gravitational field of the Earth.

(ii) The gravitational field strength, g, is uniform close to the Earth's surface. Describe the pattern of gravitational field lines close to the surface of the Earth.

(b) The planet Saturn has mass $5.7 \times 10^{26}\ kg$ and radius $6.0 \times 10^{7}\ m$.

(i) Calculate the gravitational field strength g_s at Saturn's surface.

(ii) Saturn's second-largest moon, Rhea, has orbital radius $5.3 \times 10^{8}\ m$ and mass $2.3 \times 10^{21}\ kg$. Calculate for Rhea

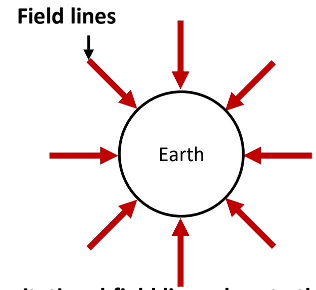
(1) its orbital speed v

(2) Its kinetic energy

Gravitational Fields

Exam Style Question 1

(a) (i) Draw lines on Fig. 2.1 to show the shape and direction of the gravitational field of the Earth.



(ii) Describe the pattern of gravitational field lines close to the surface of the Earth.

The field lines are parallel to each other, perpendicular to the surface of the Earth and are evenly spaced.

(b) (i) Calculate the gravitational field strength g_s at Saturn's surface.

Use $g = \frac{GM}{R^2}$

$$g = \frac{(6.67 \times 10^{-11}\ Nm^2kg^{-2})(5.7 \times 10^{26}\ kg)}{(6.0 \times 10^{7}\ m)^2}$$

$$g = 11\ N\ kg^{-1}$$

(ii) (1) its orbital speed v

Remember Gravitational force = centripetal force

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Therefore: $v^2 = \frac{G\,M\,m\,r}{r^2\,m}$

$$v^2 = \frac{GM}{r} = \frac{(6.67 \times 10^{-11}\ Nm^2kg^{-2})(5.7 \times 10^{26}\ kg)}{(5.3 \times 10^{8}\ m)}$$

$$v^2 = 7.173 \ldots \times 10^{7}$$

$$v = \sqrt{7.173 \ldots \times 10^{7}} = 8469.59 \ldots = 8.5 \times 10^{3}\ m\,s^{-1}$$

(ii) (2) Its kinetic energy

Use $KE = \frac{1}{2}mv^2$

$$KE = \frac{1}{2}(2.3 \times 10^{21}\ kg)(8.469 \times 10^{3}\ m\,s^{-1})$$

$$KE = 8.2 \times 10^{28} J$$

Gravitational Fields

Exam Style Question 2

(a) (i) State Newton's law of gravitation.

(ii) Define gravitational field strength, g.

(b) Titan, a moon of Saturn, has a circular orbit of radius $1.2 \times 10^6\ km$.
The orbital period of Titan is 16 Earth days.

(i) Calculate the speed of Titan in its orbit.

(ii) Show that the mass of Saturn is about $5 \times 10^{26}\ kg$.

(c) Rhea is another moon of Saturn with a smaller orbital radius than Titan. Determine the ratio

$\frac{orbital\ period\ T_R\ of\ Rhea}{orbital\ period\ T_T\ of\ Titan}$ in terms of their orbital radii r_R , and r_T .

Gravitational Fields

Exam Style Question 2

(a)(i) State Newton's law of gravitation.
Force between two points masses is proportional to the product of masses and inversely proportional to the square of the distance between them.

(ii) Define gravitational field strength, g.
Force per unit mass

(b) (i) Calculate the speed of Titan in its orbit.
Use $v = \frac{2\pi R}{T}$

$$v = \frac{2\pi(1.2 \times 10^9\ m)}{(16\ days \times 24\ hours \times 60\ mins \times 60 second)}$$
$$v = 5454.153 \ldots = 5.5 \times 10^3\ m\ s^{-1}$$

(ii) Show that the mass of Saturn is about $5 \times 10^{26}\ kg$.

Remember Gravitational force = centripetal force

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Therefore: $v^2 = \frac{G\,M\,m\,r}{r^2\,m}$

$$v^2 = \frac{GM}{r}\ so\ M = \frac{v^2 r}{G}$$
$$M = \frac{(5454 \ldots m\ s^{-1})^2 (1.2 \times 10^9\ m)}{(6.67 \times 10^{-11}\ Nm^2 kg^{-2})}$$

$$M = 5.3 \times 10^{26}\ kg$$

(c) Rhea is another moon of Saturn with a smaller orbital radius than Titan. Determine the ratio
$\frac{orbital\ period\ T_R\ of\ Rhea}{orbital\ period\ T_T\ of\ Titan}$ in terms of their orbital radii r_R , and r_T .

Remember $T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$ so $T^2 \propto r^3$

$$\left(\frac{T_R}{T_T}\right)^2 = \left(\frac{r_R}{r_T}\right)^3$$
$$\frac{T_R}{T_T} = \sqrt{\frac{r_R^3}{r_T^3}}$$

Gravitational Fields

Exam Style Question 3

(a) Define gravitational field strength.

(b) The table shows, in modern units, information that was known to physicists at the time of Isaac Newton.

position	distance r from centre of the Earth/km	gravitational field strength g due to the Earth/N kg^{-1}
surface of Earth	6.4×10^3	9.8
Moon's orbit	3.8×10^5	2.7×10^{-3}

Use the information provided in the table to

(i) state a relationship between the gravitational field strength g and the distance r and verify this relationship.

(ii) show that the mass of the Earth is about $6 \times 10^{24}\ kg$.

(iii) determine the mean density of the Earth.

Gravitational Fields

Exam Style Question 3

(a)Define gravitational field strength.

Force per unit mass at a point in a gravitational field.

(b) Use the information provided in the table to

(i) state a relationship between the gravitational field strength g and the distance r and verify this relationship.

$g \propto \frac{1}{r^2}$ and therefore obeys the inverse square law.

Hence: $gr^2 = constant$

Earth: $gr^2 = (9.8\ N\ kg^{-1})(6.4 \times 10^3\ km)^2 = 4.0 \times 10^8$

Moon: $gr^2 = (2.7 \times 10^{-3}\ N\ kg^{-1})(3.8 \times 10^5\ km)^2 = 3.9 \times 10^8$

As the constants for both the Earth and the moon are approx. the same the relationship $g \propto \frac{1}{r^2}$ holds true.

(ii) show that the mass of the Earth is about $6 \times 10^{24}\ kg$.

Use $g = \frac{GM}{r^2}$ and rearrange for M:

$$M = \frac{gr^2}{G} = \frac{(9.81\ N\ kg^{-1})(6.4 \times 10^6\ m)^2}{(6.67 \times 10^{-11} Nm^2 kg^{-2})}$$

$$M = 6.024 \times 10^{24}\ kg$$

(iii) determine the mean density of the Earth.

Use $\rho = \frac{m}{V}$, we have the mass of the Earth now we only need the volume:

$$V = \frac{4}{3}\pi r^3$$

$$V = \left(\frac{4}{3}\pi\right)(6.4 \times 10^6\ m)^3 = 1.0980 \ldots \times 10^{21}\ m^3$$

$$\rho = \frac{m}{V} = \frac{6.024 \ldots \times 10^{24}\ kg}{1.0980 \ldots \times 10^{21}\ m^3} = 5486\ kg\ m^{-3} \approx 5500\ kg\ m^{-3}$$

Gravitational Fields

Exam Style Question 4

A satellite orbits the Earth in a circular path 800 km above the Earth's surface. At the orbit of the satellite the gravitational field strength is $7.7\ N\ kg^{-1}$. The radius of the Earth is $6400\ km$.

(a) Calculate

(i) the orbital speed of the satellite

(ii) the period of the orbit of the satellite.

(b) The orbit of the satellite passes over the Earth's poles.

(i) Show that the satellite makes about 14 $orbits$ around the Earth in 24 $hours$.

(ii) The cameras on board the satellite continually photograph a strip of the Earth's surface, of width 3000 km, directly below the satellite. Determine, with an appropriate calculation, whether the satellite can photograph the whole of the Earth's surface in 24 $hours$. State your conclusion.

(c) Suggest a practical use of such a satellite.

Gravitational Fields

Exam Style Question 4

(a)Calculate

(i) the orbital speed of the satellite.

We know that the satellite is orbiting the Earth therefore it will have a centripetal acceleration which can be calculated using $a = \frac{v^2}{r}$

And rearrange for v:

$$v = \sqrt{gr} = \sqrt{(7.7\ N\ kg^{-1})(800km + 6400km)}$$
$$v = \sqrt{(7.7\ N\ kg^{-1})(7.2 \times 10^6\ m)}$$
$$v = 7450\ m\ s^{-1}$$

(ii) the period of the orbit of the satellite.

Use $v = \frac{2\pi r}{T}$ and rearrange for T

$$T = \frac{2\pi r}{v} = \frac{2\pi(7.2 \times 10^6\ m)}{7450\ m\ s^{-1}}$$
$$T = 6072\ s$$

(b) The orbit of the satellite passes over the Earth's poles.

(i) Show that the satellite makes about 14 $orbits$ around the Earth in 24 $hours$.

Remember the satellite takes 6072 s to complete one orbit so how many orbits can it complete in 24 hours. So do the below:

$$no. of\ orbits = \frac{24\ hours \times 3600\ s}{6072\ s} = \frac{86400\ s}{6072\ s}$$
$$no. of\ orbits = 14.229 \ldots \approx 14\ orbits$$

Gravitational Fields

Exam Style Question 4

A satellite orbits the Earth in a circular path $800\ km$ above the Earth's surface. At the orbit of the satellite the gravitational field strength is $7.7\ N\ kg^{-1}$. The radius of the Earth is $6400\ km$.

(a) Calculate

(i) the orbital speed of the satellite

(ii) the period of the orbit of the satellite.

(b) The orbit of the satellite passes over the Earth's poles.

(i) Show that the satellite makes about $14\ orbits$ around the Earth in $24\ hours$.

(ii) The cameras on board the satellite continually photograph a strip of the Earth's surface, of width $3000\ km$, directly below the satellite. Determine, with an appropriate calculation, whether the satellite can photograph the whole of the Earth's surface in $24\ hours$. State your conclusion.

(c) Suggest a practical use of such a satellite.

Gravitational Fields

Exam Style Question 4

(ii) Determine, with an appropriate calculation, whether the satellite can photograph the whole of the Earth's surface in $24\ hours$. State your conclusion.

We know that the camera can photograph a width of 3000km so first lets calculate the total circumference the satellite can photograph.

$$\frac{equatorial\ circumference}{width\ of\ photograph} = \frac{2\pi r}{3000\ km} = \frac{2\pi(6400\ km)}{3000\ km} = 13.4$$

Therefore the satellite need to photograph 13.4 strips to map the entire Earths surface. But in 1 orbit the satellite can capture 2 strips of the Earth e.g. from North pole to South pole and then from South pole to North pole again. This means we can half the 13.4 strips and we get:

$$\frac{13.4}{2} = 6.7\ orbits$$

But the satellite does 14 orbits in 24 hours therefore the whole of Earths surface can be photographed in 24 hours. (See video in notes to help visualise the scenario.)

(c) Suggest a practical use of such a satellite.

Weather/spy/surveying/mapping/GPS

Gravitational Fields

Exam Style Question 5

(a) State, in words, Newton's law of gravitation.

(b) Fig. 3.1 shows the circular orbits of two of Jupiter's moons: Adrastea, A, and Megaclite, M.

Use the following data in the calculations below.

orbital radius of $A = 1.3 \times 10^8\ m$

orbital period of $A = 7.2\ hours$

gravitational field strength at orbit of $A = 7.5\ N\ kg^{-1}$

orbital radius of $M = 2.4 \times 10^{10}\ m$

Calculate

(i) the mass of Jupiter

(ii) the gravitational field strength at the orbit of M

(iii) the orbital period of M.

Gravitational Fields

Exam Style Question 5

(a) State, in words, Newton's law of gravitation.

Force is proportional to the product of the masses and inversely proportional to the square of their separation.

(b) Calculate

(i) the mass of Jupiter

Use $g = \frac{GM}{r^2}$ and rearrange for M

$$M = \frac{g\,r^2}{G} = \frac{(7.5\ N\ kg^{-1})(1.3 \times 10^8\ m)^2}{(6.67 \times 10^{-11} Nm^2kg^{-2})}$$

$$\therefore M_J = 1.9 \times 10^{27}\ kg$$

(ii) the gravitational field strength at the orbit of M

Use $g = \frac{GM}{r^2}$

$$g = \frac{(6.67 \times 10^{-11}\ Nm^2kg^{-2})(1.9 \times 10^{27}\ kg)}{(2.4 \times 10^{10}\ m)^2}$$

$$g_M = 2.2 \times 10^{-4}\ N\ kg^{-1}$$

(iii) the orbital period of M.

Remember $T^2 \propto r^3$ so $\frac{T^2}{r^3} = constant$

And we can Kepler 3rd law of planetary motion:

The ratio $\left(\frac{T^2}{r^3}\right)$ is the same (i.e. constant) for all planets.

$$\therefore \frac{T_M^2}{r_M^3} = \frac{T_A^2}{r_A^3}$$

$$T_M = \sqrt{\frac{T_A^2}{r_A^3} \times r_M^3}$$

$$T_M = \sqrt{\frac{(7.2\ hours)^2}{(1.3 \times 10^8\ m)^3} \times (2.4 \times 10^{10}\ m)^3}$$

$$T_M = 1.8 \times 10^4\ hours$$

Gravitational Potential

Exam Style Question 6

Communications satellites are usually placed in a geo-synchronous orbit.

(a) State two features of a geo-synchronous orbit.

(b) The mass of the Earth $6.00 \times 10^{24}\ kg$ and its mean radius is $6.40 \times 10^{6}\ m$.

(i) Show that the radius of a geo-synchronous orbit must be $4.23 \times 10^{7}\ m$,

(ii) Calculate the increase in potential energy of a satellite of $750\ kg$ when it is raised from the Earth's surface into a geo-synchronous orbit.

(c) Satellites in orbits nearer the Earth than geo-synchronous satellites may be used in the future to track road vehicles.

(i) State and explain one reason why geo-synchronous satellites would not be suitable for such a purpose.

(ii) Give two points you would make in arguing for or against tracking road vehicles. Explain your answers.

Gravitational Potential

Exam Style Question 6

(a) State two features of a geo-synchronous orbit.

Period is 24 hours

Remains in a fixed position relative to surface of Earth

Equatorial orbit

Same angular speed as Earth

(b) (i)Show that the radius of a geo-synchronous orbit must be $4.23 \times 10^{7}\ m$,

Time period required for a geostationary orbit is $24\ h = 86{,}400\ s$

$$\omega = \frac{2\pi}{T}$$

Also $F = \frac{GMm}{r^2}$ where $F = centripetal\ force = ma = m\omega^2 r$

$$\therefore m\omega^2 r = \frac{GMm}{r^2}$$

Simplifying

$$\omega = \sqrt{\frac{GM}{r^3}}$$

Therefore: $\frac{2\pi}{T} = \sqrt{\frac{GM}{r^3}}$

$$r^3 = \frac{GMT^2}{(2\pi)^2} = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 86400^2}{4\pi^2} = 7.57 \times 10^{22}$$

$$r = \sqrt[3]{7.57 \times 10^{22}} = 4.23 \times 10^{7} m$$

(b) (ii) Calculate the increase in potential energy of a satellite of $750\ kg$ when it is raised from the Earth's surface into a geo-synchronous orbit.

Use $\Delta V = -GM\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

$$\Delta V = (6.67 \times 10^{-11}\ Nm^2kg^{-2})(6 \times 10^{24}\ kg)\left(\frac{1}{6.4 \times 10^{6}\ m} - \frac{1}{4.23 \times 10^{7}\ m}\right)$$

$$\Delta V = 5.307 \ldots \times 10^{7}\ J\ kg^{-1}$$

Now use $\Delta E_P = m\Delta V$

$$\Delta E_P = (750kg)(5.307 \times 10^{7}\ K\ kg^{-1})$$

$$\Delta E_P = 3.98 \times 10^{10}\ J$$

Gravitational Potential

Exam Style Question 6

Communications satellites are usually placed in a geo-synchronous orbit.

(a) State two features of a geo-synchronous orbit.

(b) The mass of the Earth $6.00 \times 10^{24}\ kg$ and its mean radius is $6.40 \times 10^{6}\ m$.

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(i) State and explain one reason why geo-synchronous satellites would not be suitable for such a purpose.

(ii) Give two points you would make in arguing for or against tracking road vehicles. Explain your answers.

Gravitational Potential

Exam Style Question 6

(c) (i) State and explain one reason why geo-synchronous satellites would not be suitable for such a purpose.

Signal would be too weak at large distances as the signal spreads out more the further it travels.

(c) (ii) Give two points you would make in arguing for or against tracking road vehicles. Explain your answers.

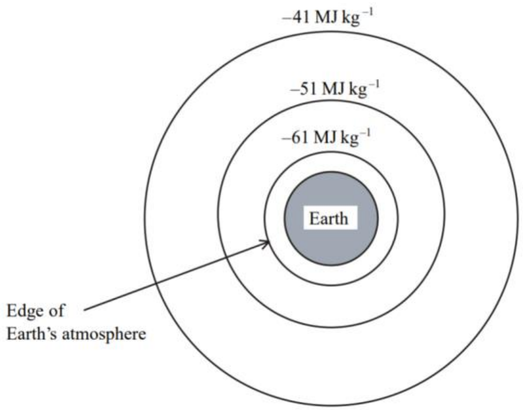
Pros: Stolen vehicles can be tracked and recovered
Uninsured or unlicensed vehicles can be apprehended

Cons: Poosibility of invasion of privacy

Gravitational Potential

Exam Style Question 7

The diagram shows three equipotential surfaces centred about the Earth with their values marked.



(a) State two deductions that can be made from the diagram.

(b) The gravitational potential at the Moon's orbit due to the Earth alone is approximately $-1.0\ MJ\ kg^{-1}$. Use this fact and information from the diagram above to show that a spacecraft, returning from the Moon's orbit using only the gravitational attraction of the Earth, would be travelling at approximately $11\ km\ s^{-1}$ on arrival at the Earth's atmosphere.

(c) There is a point between the Earth and the Moon where their gravitational attractions on a given mass are equal and opposite. Use the formula for the gravitational attraction between point masses to show that this distance is nearly 10 times further from the Earth than from the Moon.

Mass of the Earth = $6.0 \times 10^{24}\ kg$

Mass of the Moon = $7.4 \times 10^{22}\ kg$.

Gravitational Potential

Exam Style Question 7

(a) State two deductions that can be made from the diagram.

1) The gravitational potential is increasing with height therefore work must be done to move away from the Earth.
2) The field is non-uniform and so the field strength decreases with height.

(b) Show that a spacecraft, returning from the Moon's orbit using only the gravitational attraction of the Earth, would be travelling at approximately $11\ km\ s^{-1}$ on arrival at the Earth's atmosphere.

1) We know $\Delta GPE = \Delta E_P = m\Delta V$ so lets find the change in gravitational potential first:

$$\Delta E_P = -1\ MJkg^{-1} - (-61\ MJkg^{-1}) = 60\ MJ\ kg^{-1}$$

2) GPE and KE are interchangeable therefore:

$$\frac{1}{2}mv^2 = \Delta V$$

3) Cancel out the m and you get:

$$\frac{1}{2}v^2 = \Delta V$$

$$\frac{1}{2}v^2 = 60\ MJ$$

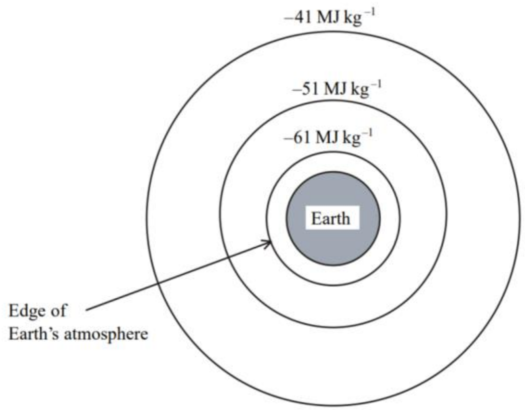
4) Rearrange for v:

$$v = \sqrt{\frac{60\ MJ}{\left(\frac{1}{2}\right)}} = 1.095 \times 10^4\ ms^{-1} \approx 11\ km\ s^{-1}$$

Gravitational Potential

Exam Style Question 7

The diagram shows three equipotential surfaces centred about the Earth with their values marked.



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(c) There is a point between the Earth and the Moon where their gravitational attractions on a given mass are equal and opposite. Use the formula for the gravitational attraction between point masses to show that this distance is nearly 10 times further from the Earth than from the Moon.

Mass of the Earth = $6.0 \times 10^{24}\ kg$

Mass of the Moon = $7.4 \times 10^{22}\ kg$.

Gravitational Potential

Exam Style Question 7

(c) Show that this distance is nearly 10 times further from the Earth than from the Moon.

We know that there is a point between the Earth and the Moon where their gravitational attractions on a given mass are equal and opposite:

$$\therefore F_{Earth} = F_{Moon}$$
$$\frac{GM_Em}{r_E^2} = \frac{GM_Mm}{r_M^2}$$

Simplifying gives:

$$\frac{M_E}{r_E^2} = \frac{M_M}{r_M^2}$$

Rearranging gives:

$$\frac{r_E^2}{r_M^2} = \frac{M_E}{M_M}$$
$$\frac{r_E^2}{r_M^2} = \frac{6.0 \times 10^{24}\ kg}{7.4 \times 10^{22}\ kg} = 81.08\ ...$$
$$\frac{r_E}{r_M} = \sqrt{81.08\ ...} = 9.0045\ ... \approx 10$$

Therefore this proves that the point between the Earth and the Moon where their gravitational attractions on a given mass are equal and opposite is nearly 10 times further from the Earth than from the Moon.

Escape Velocity

Exam Style Question 10

(a) (i) State what is meant by the term escape velocity.

(ii) Show that the escape velocity, v, at the Earth's surface is given by $v = \sqrt{\frac{2GM}{R}}$

where M is the mass of the Earth

and R is the radius of the Earth.

(b) State a reason why rockets launched from the Earth's surface do not need to achieve escape velocity to reach their orbit.

Escape Velocity

Exam Style Question 10

(a) (i) State what is meant by the term escape velocity.

Minimum speed that will allow an object to leave/escape (Earth's or) a planets gravitational field.

(ii) Show that the escape velocity, v, at the Earth's surface is given by $v = \sqrt{\frac{2GM}{R}}$

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

So:

$$v = \sqrt{\frac{2GM}{r}}$$

(b) State a reason why rockets launched from the Earth's surface do not need to achieve escape velocity to reach their orbit.

Because the rocket doesn't actually leave the gravitational field and so less energy is needed to achieve orbit than to escape from Earth's gravitational field.

Please see **'11.1.1 Gravitation and Planetary Motion notes'** pack for revision notes.

For more revision notes, tutorials and worked examples please visit **www.tutorpacks.co.uk.**