A2 Level Physics

Chapter 18 – Electric Fields

18.1.1 Electric Fields

Notes

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Electric Field

Electric fields can be attractive or repulsive.

An electric field is a region which exists around particles and objects which have an electric charge. This electric field is a region where the charged particle or object can attract or repel other charges.

Electric charge, Q, is measured in coulombs (C) and can be positive or negative. Oppositely charged particles attract each other, while like charges repel one other. A charged object will experience a force if it is placed in an electric field.

Electric Field Strength

Electric field strength (E), is defined as the force per unit positive charge.

It's the force that a $+1C$ charge would experience if it were placed in an electric field.

$$E = \frac{F}{Q}$$

Where:

- E = electric field strength in NC^{-1}
- F = force on the charged object in N
- Q = charge of the object in C

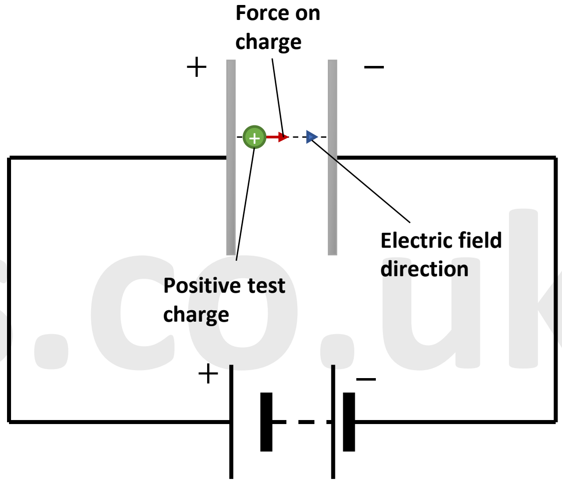
Electric field strength (E) is a vector quantity and so $E - values$ must be considered vectorially (that is, direction as well as magnitude should be taken into consideration) when calculating the resultant E due to multiple charges.

Electric Field Lines

Field lines (also known as Lines of Force) are used to visually represent an electric field.

When a positive test charge is placed in the field, the electric field lines indicate which direction it will move.

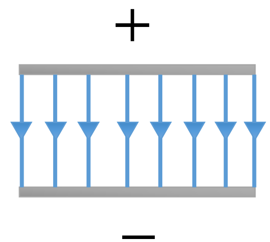
The positive test charge is forced to move from left to right by the electric field between the plates as it is repelled by the positively charged plate on the left and attracted by the negatively charged plate on the right.



A uniform electric field is represented by parallel, equidistant field lines.

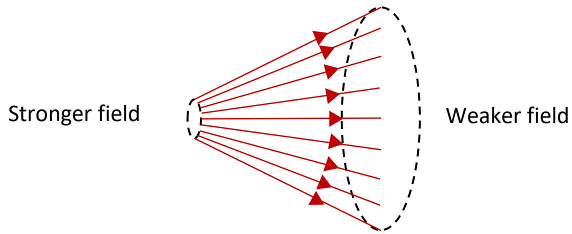
This shows that the magnitude and direction of the electric field are both constant.

Now parallel plates, won't necessarily have a positive plate and a negative plate. The field lines (for parallel plates) are drawn from the plate with the higher positive voltage to the plate with the lower positive voltage. In this scenario, we'll just assume the plate with the lowest positive voltage is the negative plate.

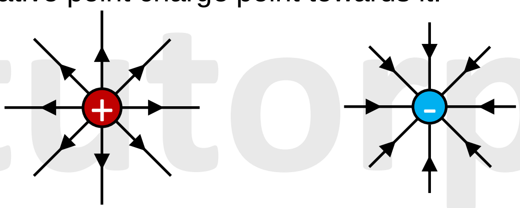


Electric Field Lines

A non-uniform electric field is represented by non-parallel field lines, with diverging field lines indicating a weakening field and converging field lines indicating a strengthening field.



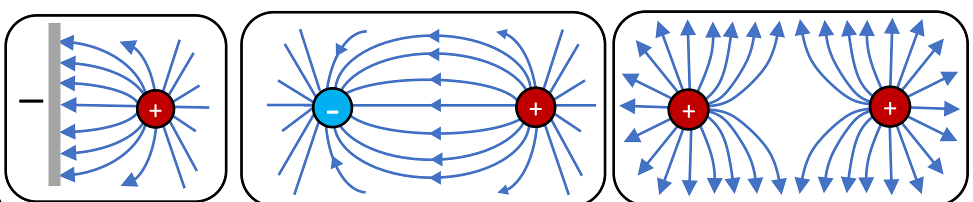
A radial field exists for point charges. The field lines diverge, indicating that the strength of the electric field weakens with distance. The field lines of a positive point charge point away from it, while those of a negative point charge point towards it.



The diagrams below show:

a) A field that exists between a positively charged metal plate and a negatively charged metal plate.

b) The field that exists between opposite charges.

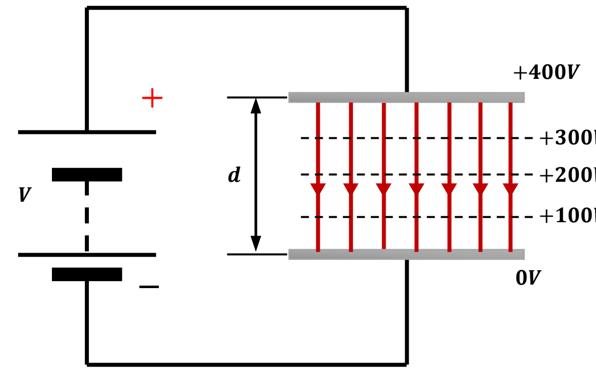


Uniform electric field

A uniform electric field is formed between two parallel metal plates separated by a distance (d) with a potential difference (V) between them (as shown below).

The field lines are:

- Parallel to each other
- At right angles to the plates
- Directed from the positive plate to the negative plate



The field strength (E) is increased by:

- Increasing the potential difference (V)
- Decreasing the plate separation (d)

The field strength E is constant at all points between the two plates and is calculated using:

$$E = \frac{V}{d}$$

Where:

E = electric field strength in Vm^{-1} or NC^{-1}

V = potential difference between the plates in V

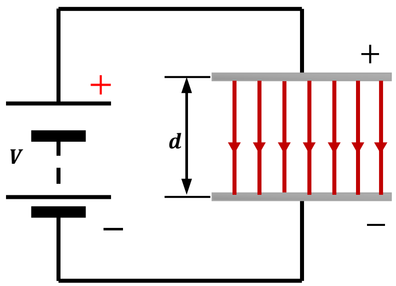
d = distance between the plates in m

It's worth noting that the potential difference between the plates will be equal to the top plate's potential. This is because the bottom plate's potential in this example is 0V. However, this won't always be the case, so you might find it easier to think of it as ΔV.

Permittivity

A capacitor consists of two charged plates parallel to one another. If we take a closer look at the previous diagram, we can see two parallel plates, one with a positive charge and the other with a negative charge. We can calculate the capacitor's capacitance using the formula:

$$C = \frac{\varepsilon A}{d}$$



Where:

C = capacitance in farads, F

A = effective area of a plate in m^2

d = distance between the plates, d

ε = permittivity of material (see below)

We can replace ε with ε_0 if the gap between the two plates is air :

$$C = \frac{\varepsilon_0 A}{d}$$

Relative Permittivity, ε_r

A capacitor's properties affect how much charge it can store at a given voltage (its capacitance). The dielectric material (insulators) that separates the two conducting plates is one of the things you can change. This changes the capacitance because different material have different relative permittivities.

Permittivity is a measure of how difficult it is to generate an electric field in a medium. The higher a material's permittivity, the more charge is required to generate an electric field of a given size.

Permittivity

Relative permittivity is the ratio of the permittivity of a material to the permittivity of free space:

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

Where:

ε_r = relative permittivity of material one

ε = permittivity of material one in Fm^{-1}

ε_0 = permittivity of free space = $8.55 \times 10^{-12} Fm^{-1}$

Therefore, capacitance can be calculated using:

$$C = \frac{\varepsilon A}{d} = \frac{\varepsilon_r \varepsilon_0 A}{d}$$

Uniform Electric Field

Note:

When a charge (Q) is placed in a uniform electric field of strength (E) between the plates, the force (F) acting on the charge is given by:

$$F = QE = \frac{QV}{d}$$

Where:

E = electric field strength in NC^{-1} or Vm^{-1}

F = force on the charged object in N

Q = charge of the object in C

V = potential difference between the plates in V

d = distance between the plates in m

The following explains why electric field strength is measured in two units:

$$F = QE = \frac{QV}{d}$$

Rearranging the equation gives:

$$\frac{F}{Q} = \frac{V}{d} = E$$

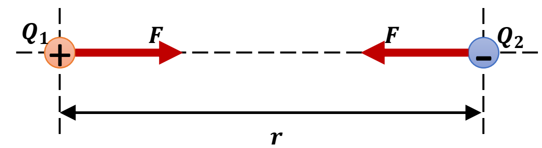
$$NC^{-1} = Vm^{-1}$$

Coulomb's Law

According to Coulomb's law any two charged particles exert a force on each other that is proportional to the product of their charges and inversely proportional to the square of their separation.

For two charges, Q_1 and Q_2, distance (r) apart, in a vacuum (or air), the force (F) which they exert on each other is:

$$F \propto \frac{Q_1 Q_2}{r^2}$$



$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$$

Where:

F = force on the object in N

Q_1 and Q_2 = charges of the two objects in C

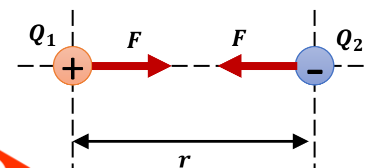
ε_0 = "epsilon-nought", the permittivity of free space $(= 8.85 \times 10^{-12} Fm^{-1})$ in farads per metre (Fm^{-1})

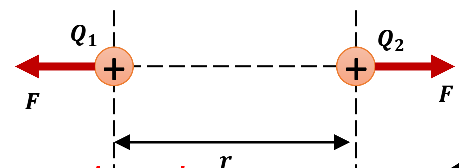
r = distance between Q_1 and Q_2 in m

The force on Q_1 is always equal and opposite to the force on Q_2 - the direction of the force is determined by the charges.

The force is attractive when the charges are opposite.

The force is repulsive when the charges are the same.





Coulomb's Law

Note:

- The inverse square law applies to the Coulomb's law, therefore $F \propto \frac{1}{r^2}$. So, the force between the charges is weaker the further apart they are.
- $\frac{1}{4\pi\varepsilon_0}$ is the constant of proportionality = $8.99 \times 10^9\ Nm^2C^{-2}$.
- ε_0 is the permittivity of a vacuum = $8.85 \times 10^{-12} Fm^{-1}$.

If the point charges aren't in a vacuum, the magnitude of the force F is also determined by the material's permittivity, ε. When using Coulomb's law, air can be considered as a vacuum.

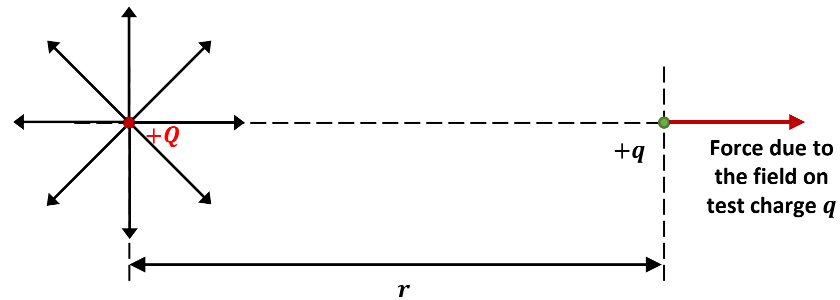
If the electric field is not in air or a vacuum, you'd replace ε_0 with the ε of the material in Coulomb's law.

If the question doesn't specify otherwise, assume the charges are in a vacuum.

Electric Field Strength for a point charge

A point charge is an expression used for a charged object where the distances under consideration are much greater than the size of the object.

A test charge in an electric field is a point charge that does not change the electric field in which it is placed.



Consider the electric field produced by a point charge $+Q$ as shown in the diagram above.

The force (F) acting on a test charge $+q$ placed at a distance (r) from $+Q$ is:

$$F = \frac{Qq}{4\pi\varepsilon_0 r^2}$$

And since $E = F/q$ (By definition), the electric field strength at distance r from Q is:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Where:

E = electric field strength in NC^{-1}

Q = point charge in C

ε_0 = the permittivity of free space = $8.85 \times 10^{-12} Fm^{-1}$

r = distance from the point charge in m

If Q is negative, the above calculation returns a negative value of E, which corresponds to field lines pointing inwards towards Q.

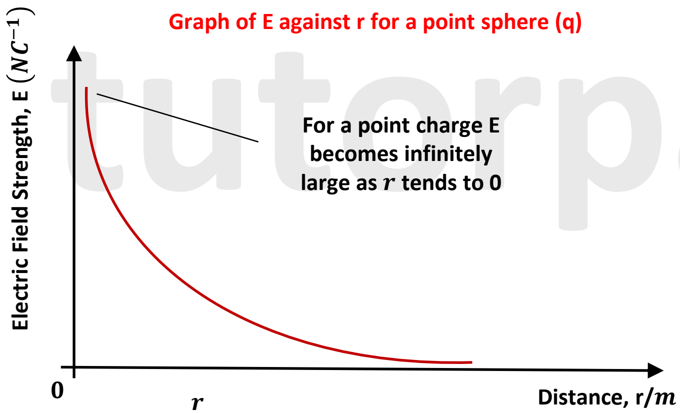
Electric Field Strength for a point charge

In a radial field, the electric field strength, E, depends on the distance r from the point charge Q.

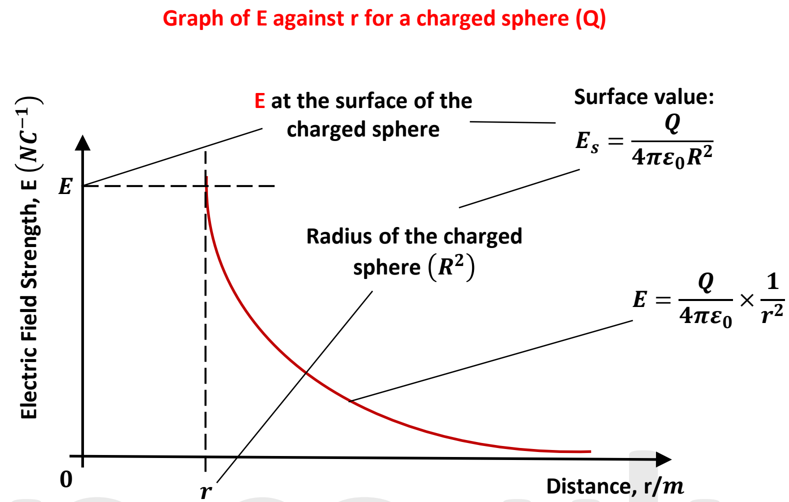
Note:

This is another example of the inverse square law $E \propto \frac{1}{r^2}$. As you get further away from Q, the strength of the field decreases.

The field lines of a radial field get further apart as the distance increases, and so if you plot the electric field strength against r you get:



Electric Field Strength for a point charge



E against r graph usually only show the magnitude of E (thus they're always positive). However, if E is negative, the graph would just be reflected in the horizontal axis.

Electric Potential and Energy

Electric potential (V) is the electrical potential energy per unit charge $\left(\frac{Joules}{Coulomb} = \frac{J}{C}\right)$ at a point in a field.

Or

Electric potential (V) at a point in a field is the energy required to move a unit positive charge from infinity to a point in a field. At infinity the electric potential will be zero (see the graph below).

Point charges:

The work done on an object with a unit charge is the force acting on the charge multiplied by distance travelled.

Therefore: $W = Fd$

However from Coulomb's law states that:

$$F = \frac{Qq}{4\pi r^2 \varepsilon_0}$$

Therefore:

$$W = F \times d = \frac{Qq}{4\pi\varepsilon_0 r^2} \times r$$

So, to move a point charge q from infinity to a radius r, the work done and thus the electrical potential energy required is:

$$E_e = \frac{Qq}{4\pi\varepsilon_0 r}$$

With the electrical potential energy defined as E_e to distinguish it from the electric field.

But remember, the electric potential is defined as the electrical potential energy per unit charge, therefore:

$$V = \frac{E_e}{q}$$

So we get:

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

Where:

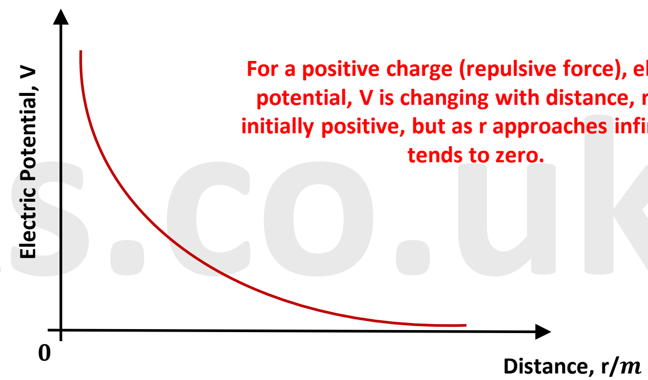
- V = electric potential in J/C
- Q = charge creating the electric field in C
- r = distance from the charge in m
- ε_0 = the permittivity of free space = $8.85 \times 10^{-12} Fm^{-1}$

Electric Potential and Energy

The electric potential of a point depends on how far it is from the charge creating the electric field and the size of that charge.

The charge Q determines the sign of V. When Q is positive and the force is repulsive, V is positive; when Q is negative and the force is attractive, V is negative.

The magnitude of V is greatest on the surface of the charge and decreases as the distance from the charge increases; V will be zero at an infinite distance from the charge.



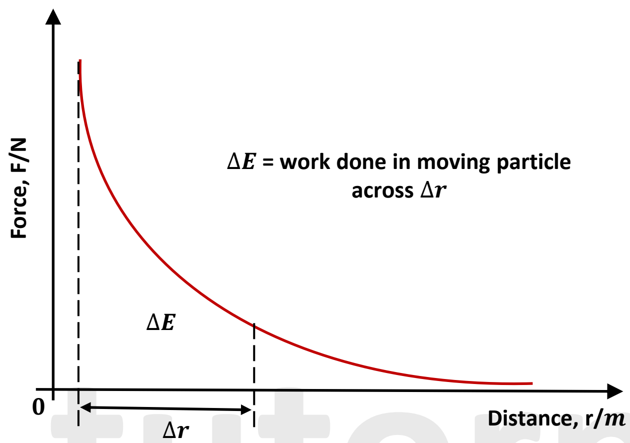
The graph would be reflected on the horizontal axis if V changed with r for a **negative charge (attractive force).**

The gradient of a tangent to the graph gives the field strength E at that point:

$$E = \frac{\Delta V}{\Delta r}$$

Force-distance graphs

The following graph shows the force on a charge in a radial field against distance:



This graph shows a repulsive field; an attractive field would be a reflection in the x-axis.

The area under this graph is:

$$force \times distance = work\ done$$

Electric Potential Energy

Electric potential energy at a point is the work done to move a charge, q, from infinity to the point in field.

As electric potential equals the work done per unit charge, we may calculate electric potential energy by multiplying this value by q:

$$E = Vq$$

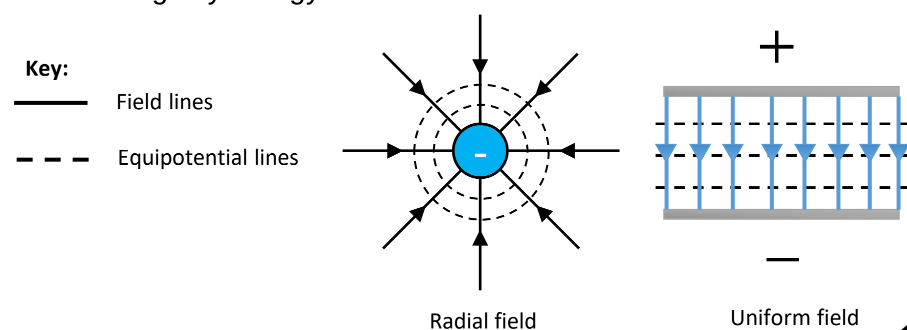
$$E = \frac{Qq}{4\pi\varepsilon_0 r}$$

If the charges Q and q are oppositely charged, the electrical potential energy will be negative because q will gain KE as it accelerates from infinity towards the charge Q. This indicates that it would take energy to separate the particles, and the PE's value would be the amount of energy required to move the particles q from a distance r away from the Q to infinity.

EQUIPOTENTIALS

Equipotentials exist in electric fields, just as they do in gravitational fields. The field lines are always perpendicular to the equipotential lines, just as they are in gravitational fields. This means that the equipotentials for a point charge are spherical surfaces, but the equipotentials for parallel plates are flat planes.

Remember that travelling along an equipotential no work is done; so an electric charge can travel along an equipotential without transferring any energy.

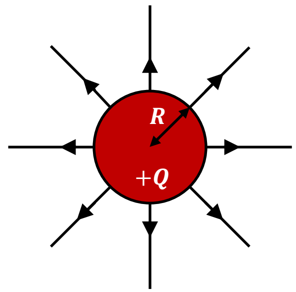


Capacitance of a Sphere

A charged sphere will have a charge that is evenly distributed across its surface. By modelling the sphere as a point charge, we can determine its capacitance.

Remember the electric potential is found at the sphere's surface:

$$V = \frac{Q}{4\pi\varepsilon_0 R}$$



Where:

R = radius of the charged sphere in m

The following equation is for capacitance:

$$C = \frac{Q}{V}$$

It's worth noting that the potential difference, V, is identical to the electrical potential, V.

Therefore we can substitute the equation for the electric potential into the equation for the capacitance:

$$C = \frac{Q}{\left(\frac{Q}{4\pi\varepsilon_0 R}\right)}$$

$$C = 4\pi\varepsilon_0 R$$

Similarities and Difference between Electric and Gravitational Fields

	Electric Field	Gravitational Field
Similarities		
Obey the inverse square law of force	Coulomb's law of force $F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$	Newton's law of gravitation $F = \frac{G m_1 m_2}{r^2}$
Have Uniform Fields	E is constant field lines are parallel	g is constant field lines are parallel
Have Radial fields	Due to a point charge (Q) $E = \frac{Q}{4\pi\varepsilon_0 r^2}$	Due to a point mass (M) $g = -\frac{GM}{r^2}$
Differences		
Action	Between any two charged objects	Between any two masses
Type of Force	Unlike charges attract Like charges repel	Attraction only
Constant of proportionality	$\frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 Nm^2C^{-2}$	$G = 6.67 \times 10^{-11} Nm^2kg^{-2}$

Please see '18.1.2 Electric Fields worked examples' pack for exam style questions.

For more revision notes, tutorials and worked examples please visit www.tutorpacks.co.uk.