A2 Level Physics

Chapter 12 – Space

12.3.1 Cosmology

Notes

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Huge Distance Units

When you leave Earth and look at the distances between objects in space, you rapidly realise how enormous those distances are.

Even though the Moon is our closest neighbour, it is about $384{,}400{,}000\ m$ away. The Sun is $147{,}460{,}000{,}000\ m$ away, and Mars is $546{,}000{,}000{,}000\ m$ away. As you can see, taking measurements across the universe produces massive numbers. Although using standard form notation helps, astronomers have come up with a number of alternative units for distance in order to reduce the magnitude of the numbers involved.

The Light Year

You've probably heard of the term "light years." Light years are a unit of measurement for distance rather than time. The unit is used to measure the distance between stars and galaxies.

In a vacuum, a light year is the distance travelled by light in a year.

$$1\ light\ year = (2.9979 \times 10^{8})(365.25)(24)(3600)$$

$$\therefore \mathbf{1\ light\ year = 9.461 \times 10^{15} m}$$

The Astronomical Unit (AU)

This unit is used to measure the distances within the solar system.

The astronomical unit (AU) is the mean radius of the Earth's orbit around the Sun.

$$\mathbf{1AU = 1.496 \times 10^{11}\ m}$$

Since the Earth's orbit around the Sun is elliptical, the mean distance $(1.496 \times 10^{11}\ m)$ is calculated from the radius of the Earth's orbit around the Sun in January $(1.471 \times 10^{11}\ m)$ and in July $(1.521 \times 10^{11}\ m)$.

The Parsec (pc)

The parsec (pc) is another unit used to measure distances between stars and galaxies, but we must first understand stellar parallax and the arcsecond before we can define the parsec (pc).

Stellar Parallax

In July Earth is at position 1 in its orbit around the Sun. A close Star X is photographed against the background of stars at a distant. At this position, Star X appears to be in the position of A.

A photograph of Star X is taken exactly 6 months later (in January). The Earth is in position 2 in its orbit around the Sun in January. However, it seems that Star X has moved to position B at this position.

The apparent shift in position of Star X is called stellar parallax.

The distance to Star X can be calculated based on how it shifted in comparison to other stars that are so far away that they don't appear to move at all (background stars). This is accomplished by comparing the location of Star X in reference to background stars at various points throughout the Earth's orbit.

You can see parallax in action by extending your palm out in front of your face and closing your right eye first, then your left. In respect to the background, your hand will appear to move. The closer your hand is to your face, the more it moves in relation to the background.

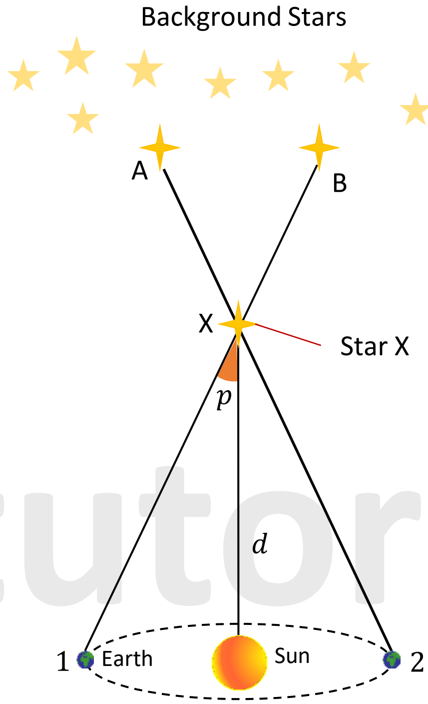
The parallax angle (p) is used to measure parallax. If you observe the position of Star X at either end of the Earth's orbit (6 months apart), the angle of parallax is half the angle that Star X makes when it moves from position A to position B (i.e. the angle made between $1X2$ in the diagram). The closer the star is to you, the larger the angle.

The parallax angle (p) is much smaller than 1 degree so we measure it in arcseconds, a much smaller unit of angular measure.

$$1\ arcsecond = \frac{1}{3600}\ degrees$$

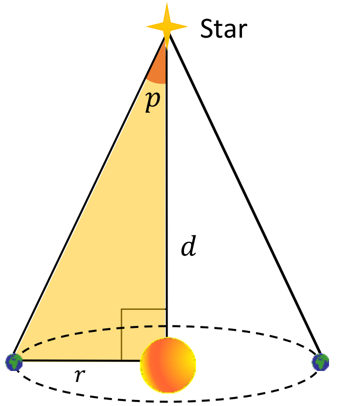
Very distant stars appear not to move as we orbit the Sun because the angle by which they move is too small for us to measure.

The Parsec (pc)



The Parsec (pc)

By measuring the angle of parallax and knowing the diameter of the Earth's orbit, the distance of a nearby star can be estimated.



You can calculate the distance to the nearby star, d, by using trigonometry on the shaded triangle above:

$$tan\theta = \frac{r}{d}$$

$$\therefore d = \frac{r}{\tan p}$$

For small angles $\tan p \approx p$, where p is in radians. You may assume this for parallax calculations (if you're working in radians) because the angles used in astronomy are so small. So:

$$d \approx \frac{r}{p}$$

Where:

d = distance to the star

r = radius of the Earth’s orbit

p = angle of parallax in radians

Remember, the angle in radians = angle in degrees × $\frac{\pi}{180}$.

The Parsec (pc)

A star is exactly one parsec (pc) away if the angle of parallax is $1\ arcsecond = \left(\frac{1}{3600}\right)°$.

This information can be used to calculate the value of 1 parsec in metres.

For very small angles we know:

$$d = \frac{r}{p}$$

r = radius of the Earth's orbit = $1AU = 1.496 \times 10^{11}\ m$

$$p = angle\ of\ parallax\ = 1\ arcsecond = \frac{1}{3600}° = 4.848 \times 10^{-6} rads$$

Therefore:

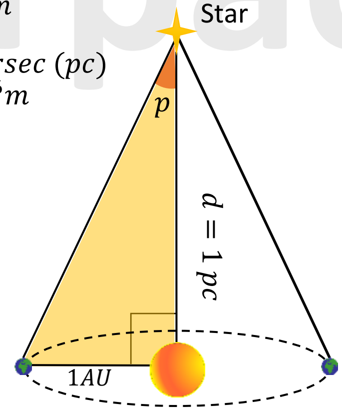
$$d = \frac{1.496 \times 10^{11}}{4.848 \times 10^{-6}}$$

So:

$$d = 3.086 \times 10^{16} m$$

This is the value for $1\ Parsec\ (pc)$

$$1\ pc = 3.086 \times 10^{16} m$$



The Parsec (pc)

There is a slightly faster method of calculating the distance to a nearby star. If the distance to the star (d) is measured in parsecs and the angle of parallax (p) is measured in arcseconds, the following equation can be used:

$$d = \frac{1}{p}$$

Standard Candles

A standard candle is a source with a known luminosity. Luminosity is the radiant power output (or the amount of light emitted by an object per unit time). Luminosity is measured in watts (W) or solar luminosities $(L\odot)$.

$$1\ L\odot = 3.8 \times 10^{26}\ W$$

In other words Luminosity is equal to power (P).

For distances that are to large to be measured by parallax, astronomers use 'standard candles.'

Remember further away light sources appear fainter because the light is spread out over a greater area.

This is because intensity is defined as the power (P) per unit cross-sectional area (A).

$$I = \frac{P}{A}$$

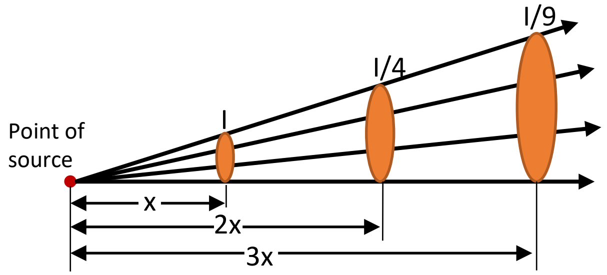
Where:

I = intensity of the radiation in Wm^{-2}

P = Power in Watts ,W

A = cross-sectional area in m^2

The inverse-square law is obeyed when the light source is spreading out from a point source (as indicated in the image below):

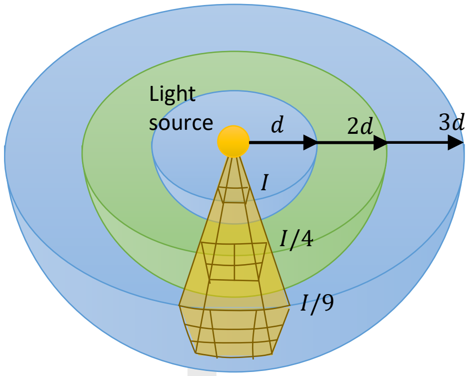


Standard Candles

We can estimate a source's distance based on how bright it appears from Earth if we know its luminosity.

We will have to make use of: $I = \frac{P}{A}$

According to the inverse square law, a star's energy will spread out in all directions across the surface of an ever-increasing sphere. To put it another way, the light that reaches Earth has been spread out over a sphere. Since a sphere's surface area is $4\pi r^2$, we can use the following equation to calculate the radiant energy intensity at a given distance, d, from a star:



$$I = \frac{L}{4\pi d^2}$$

Where:

- I = intensity of the radiation in Wm^{-2}.
- L = luminosity in watts, W.
- d = distance from the source (star) in m.

Standard Candles

Worked Example

The luminosity of Betelgeuse is $5.4 \times 10^{30}\ W$. Its radiant energy intensity at the Earth is $1.1 \times 10^{-8}\ W\ m^{-2}$. How far away is Betelgeuse?

$$I = \frac{L}{4\pi d^2}$$

$$d^2 = \frac{L}{4\pi I}$$

$$d = \sqrt{\frac{L}{4\pi I}} = \sqrt{\frac{5.4 \times 10^{30}}{4\pi(1.1 \times 10^{-8})}}$$

$$d = 6.2 \times 10^{18}\ m$$

Standard Candles

Continue to the next page.

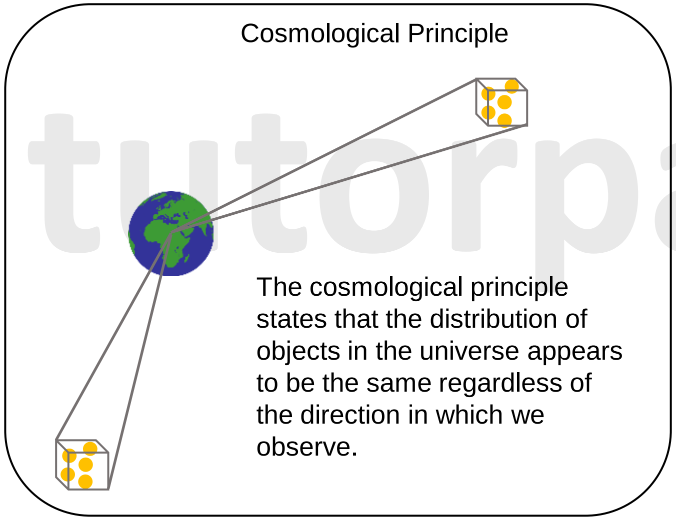
Cosmological Principle

The cosmological principle states that the Universe is homogeneous and isotropic on a large scale.

This means on a large scale, the universe is UNIFORM and as long as a large enough volume is used, the universe is:

- HOMOGENEOUS – matter is uniformly distributed (density is constant),
- ISOTROPIC – the universe is the same in all directions to every observer, and it has no centre or edge, and

Therefore, no matter where we look in the Universe, we will see the same distribution of objects.



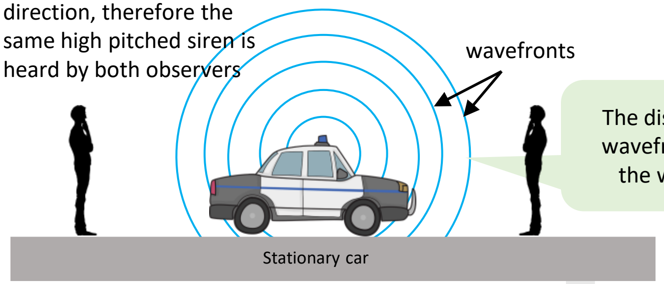
The cosmological principle states that the distribution of objects in the universe appears to be the same regardless of the direction in which we observe.

The cosmological principle also implies that the laws of physics are universal. So the same physical laws and models that apply on Earth apply to faraway stars, galaxies, and the entire Universe.

Doppler Effect

Consider a police car driving by. Its siren is higher-pitched as it is travelling towards you, and then suddenly after the car has passed by, the pitch of the siren is lower. This change in frequency and wavelength as the source or observer moves towards (or away from) each other is called the Doppler effect.

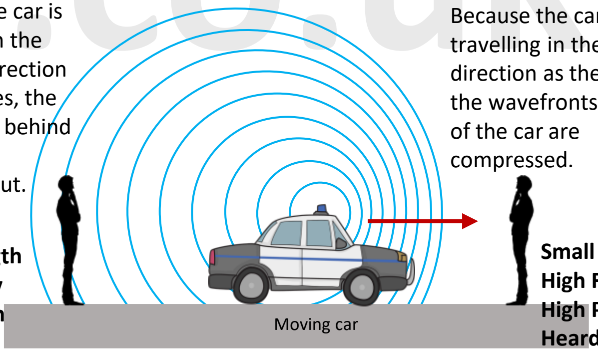
Same frequency in all direction, therefore the same high pitched siren is heard by both observers



The distance between wavefronts is equal to the wavelength, λ.

Because the car is travelling in the opposite direction of the waves, the wavefronts behind the car are stretched out.

Because the car is travelling in the same direction as the waves, the wavefronts ahead of the car are compressed.



Long wavelength
Low Frequency
Low Pitch Siren
Heard

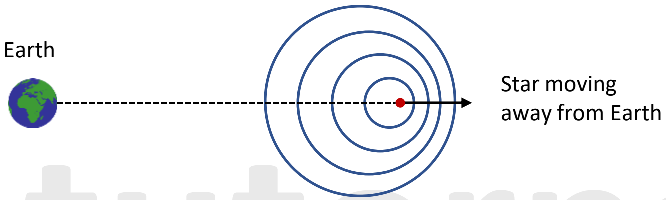
Small Wavelength
High Frequency
High Pitch Siren
Heard

The frequency and the wavelength change because the waves bunch together in front of the source and stretch out behind it. The amount of stretching or bunching together is determined by the source's velocity.

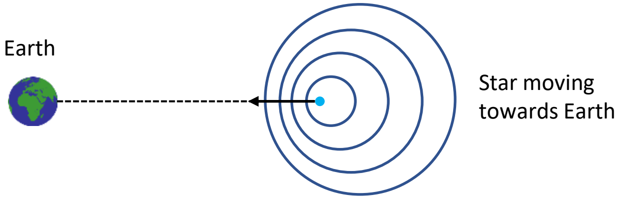
The Doppler Redshift

All waves, including electromagnetic radiation, are affected by the Doppler effect.

According to the Doppler effect, as a star or galaxy moves away from Earth (i.e. recedes), the wavelength of light from that star or galaxy becomes longer and the frequency becomes lower. This shifts the light that we receive towards the red end of the EM spectrum and is called a red shift. This means that the light we get from a star that is travelling away from us is redder than the star's actual light.



When a star or galaxy is moving towards Earth (i.e. approaching) the wavelength shortens and the frequencies increases. This shifts the light that we receive towards the blue end of the EM spectrum and is called the blue shift.



The Doppler Redshift

On the EM spectrum, on the visible light region, this effect can be seen.

The line spectra of light from receding stars and galaxies have been shown to be slightly shifted towards the RED end of the spectrum (RED-SHIFTED).

The line spectra of light from approaching stars and galaxies have been shown to be slightly shifted towards the BLUE end of the spectrum (BLUE-SHIFTED).

Moving towards Earth – Blue shift

At rest

Moving away from Earth – Red shift

The amount of red (or blue) shift is determined by the speed with which the star moves away from (or towards) us. The higher the velocity, the more the waves are shifted.

The Doppler Redshift

The amount of red shift, z, shown by a galaxy allows us to calculate its speed. This can be obtained by measuring changes in wavelength or frequency.

$$z = \frac{\Delta\lambda}{\lambda} \approx \frac{\Delta f}{f} \approx \frac{v}{c}$$

Where:

- z = red shift
- $\Delta\lambda$ = difference between emitted and observed wavelengths in metres, m
- λ = original wavelength of light emitted by the source in metres, m.
- Δf = difference between emitted and observed frequencies in Herts, Hz
- f = original frequency of the light emitted from the source in Herts, Hz
- v = the speed of the source in ms^{-1}
- c = the speed of light in a vacuum in ms^{-1}

$v > 0$ indicates that the source is moving away, and so the red shift is positive.

$v < 0$ indicates the source is moving towards the observer, and the red shift is negative. Therefore the radiation is blue shifted.

An increase in wavelength is always referred to as a red shift. A decrease in wavelength is always referred to as a blue shift.

Hubble’s Law

Edwin Hubble discovered that the universe is expanding in the 19th century when he noticed that the red shift of galaxies indicated that galaxies around us were moving away from us.

The amount of red shift determines the recessional velocity or how fast the galaxy is moving away.

Hubble also discovered that a galaxy's red shift is related to its distance from us, meaning that the more distant a galaxy is, the faster it moves.

To put it differently, if you plot recessional velocity against distance, you'll see that they're proportional, implying that the universe is expanding. This finding gives rise to Hubble’s Law which states:

The further away a galaxy is, the faster it’s travelling away from us.

OR

The recession speed of a galaxy is directly proportional to its distance from Earth.

This can be expressed mathematically:

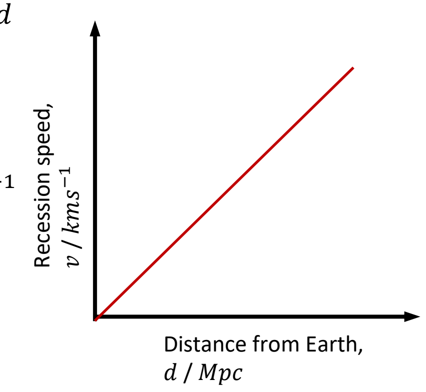
$$v = Hd$$

Where:

v = recessional velocity in kms^{-1}

H = Hubble’s constant in $kms^{-1}Mpc^{-1}$

d = distance in Mpc



Hubble's Law

Hubble's constant

The gradient of the graph of recession speed (v) against distance from Earth (D) is $\frac{v}{D}$ $(= H)$.

The Hubble's Constant, H, is difficult to calculate precisely because:

- The most accurate values of distance and recession speeds are those for relatively near stars and galaxies.
- The value of recession speed measured using Doppler shift is affected by the fact that stars and galaxies rotate as well as recede.

$H = 70 \pm 2\ kms^{-1}Mpc^{-1}$ is the most recent Hubble constant value. Over the years, the value has varied between 50 and $100\ kms^{-1}Mpc^{-1}$.

The unit of H can be converted to SI units as follows:

$$1\ kms^{-1} = 10^3\ ms^{-1}$$

$$1Mpc = 10^6 pc \times 3.086 \times 10^{16} m\, pc^{-1} = 3.086 \times 10^{22} m$$

$$\therefore H = 70kms^{-1}Mpc^{-1} = \frac{70 \times 10^3 ms^{-1}}{3.086 \times 10^{22} m} = 2.3 \times 10^{-18} s^{-1}$$

So:

$$H = 70\ kms^{-1}Mpc^{-1} = 2.3 \times 10^{-18} s^{-1}$$

Hubble's Law

Age of the universe

Hubble showed that the universe as a whole is expanding, meaning that the distance between all points in the universe is steadily growing. This indicates that the speed of recession between any two points must be proportional to the distance between them, with H being the proportionality constant (Hubble's constant).

H is a the unit of measurement used to describe the expansion of the universe and it has the same value at all points in the universe.

Hubble's law is used to calculate the distance to galaxies at the edge of the observable universe.

So, if H is a property of the expansion and the universe has been expanding at the same rate throughout its existence, knowing the Hubble constant will tell us how quickly the Universe is expanding, but we can also work backwards and determine when the Universe began.

The time taken, T, for an object to travel a distance, d, from the beginning of time at a speed of v can be calculated using the basic equation for speed:

$$speed = \frac{distance}{time}$$

$$v = \frac{d}{T}$$

$$\therefore T = \frac{d}{v} \dots\dots\dots (1)$$

If we look at the Hubble graph's gradient, $H = \frac{v}{d}$

Therefore $\frac{1}{H} = \frac{d}{v}$, and we can substitute this back into equation (1) to get the age of the universe:

$$T = \frac{1}{H}$$

Hubble's Law

Age of the universe

So if $H = 70\ kms^{-1}Mpc^{-1} = 2.3 \times 10^{-18}s^{-1}$

Therefore $T = \frac{1}{H} = \frac{1}{2.3\times10^{-18}s^{-1}} = 4.3 \times 10^{17}s \approx 13.8\ billion\ years$

Hence the universe is 13.8 billion years.

We were extremely close to the estimated age of the universe, which is 13.75 billion years.

This figure is slightly high because of the following approximations have been made:

- Throughout its existence, the galaxy has travelled at the same speed. In practise, this isn't true because it must have gained some GPE and hence lost some of its initial KE. This suggests that the galaxy's current speed is lower than its average speed, and thus if the latter had been used in the calculation, the age of the universe would have been lower.
- The value of the Hubble constant H is not precise, which means that the value obtained for time (T) is also not precise.
- The galaxies created a long time after the big bang has not been taken into consideration and this time delay has been ignored in the calculation.

For all of the reasons stated above, it is preferable to say:

$$Age\ of\ the\ universe \approx \frac{1}{H}$$

The Big Bang Theory

The universe is expanding and getting colder (because the universe is a closed system and the energy and momentum are conserved). Therefore the universe must have been denser and hotter in the past. If you travel far enough back in time you will have a hot Big Bang.

The Big Bang theory:

The universe began as a very hot and dense point (perhaps an infinitely hot and dense point) and has been expanding ever since.

This theory is support by the following observational evidence:

- The universe is expanding.
- According to Hubble's law, all galaxies are moving away from us.
- Cosmic microwave background radiation (CMBR) is widespread across the universe. According to the Big Bang theory, a lot of EM radiation was created in the early universe. Since it has nowhere else to go, this radiation should be observable even today. The wavelengths of this cosmic background radiation have been stretched as the universe has expanded, and they are now in the microwave region, hence the name CMBR. The cosmological principle is supported by the fact that the CMBR is mostly isotropic and homogeneous. However, this hypothesis is unreliable since we can't verify it experimentally because we can't recreate the Big Bang's initial conditions.
- The universe's temperature is $2.7\ K$, indicating that it is cooling.

Evolution of the Universe

The timeline from the Big Bang to the Present day is shown in the table below.

Time	Temperature	What Happened
0	Infinite	- The universe is infinitely small, infinitely dense and extremely hot.
$10^{-35}\ s$	$10^{25} K$	- The universe cooled as it expanded rapidly. - All four forces were unified: gravitational, electromagnetic, string nuclear, and weak nuclear.
$10^{-16}\ s$	$10^{14} K$	- The universe consisted of energetic quarks and leptons. - There was more matter than antimatter in the universe.
$10^{3}\ s$	$10^{12} K$	- The strong nuclear forces became dominant, combining quarks to produce hadrons (including protons).
$10^{2}\ s$	$10^{7} K$	- Helium nuclei are created because of fusion reactions between protons and neutrons. - Helium makes about 25% of the observable universe.
$10^{5}\ years$	$10^{4} K$	- Electrons combined with nuclei to form hydrogen and helium atoms.
$10^{6}\ years$	$6000 K$	- The gravitational forces dominant. Hydrogen and helium combine to produce stars, which then cluster together to form galaxies.
$10^{9}\ years$	$17 K$	- The gravitational collapse of stars resulted in the formation of heavy elements.
13 billion years	$2.7 K$	- The present time - The universe is saturated with CMBR. - This background microwave radiation on Earth is isotopic.

Evolution of the Universe

Summary:

1) At the start it was very hot and extremely dense.
2) All forces were unified.
3) Expansion led to cooling.
4) At $10^{6}\ s$ / $10^{14} K$ universe consisted of energetic quarks and leptons.
5) More matter than antimatter.
6) Strong nuclear forces become dominant and quarks combine to form hadrons/protons/neutrons.
7) Imbalance of neutrons and protons.
8) Fusion reactions between protons and neutrons produce helium.
9) Atoms formed.
10) Gravitational force become dominant and responsible for formation of stars and galaxies.
11) Temperature becomes $2.7K/3K$ and the universe is saturated with cosmic microwave background radiation.

Critical Density (ρ_c) of the Universe

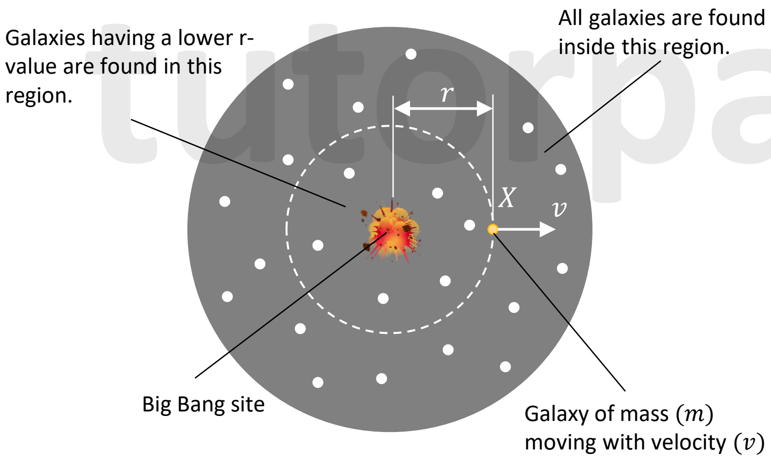
Cosmologists are asking the following question:

"Do the galaxies have enough kinetic energy to expand forever, or will they exhaust all of their kinetic energy and be pulled back to the initial singularity point in a 'big crunch' some time in the future?"

And the answer to this question is highly dependent on the universe's mean density.

Equation for the critical density (ρ_c) of the universe

The diagram below shows a galaxy X with mass (m), velocity (v), and distance (r) from the big bang site.



Critical Density (ρ_c) of the Universe

Between X and the big bang site, the total mass of all other galaxies is equal to M.

To travel outwards forever, galaxy X must overcome the attractive forces of the galaxies behind it, i.e. gain gravitational potential energy (GPE), which is supplied by:

$$GPE = \frac{GMm}{r}$$

Where: $M = V\rho = \frac{4}{3}\pi r^3 \rho$

$$GPE = \frac{G \times \frac{4}{3}\pi r^3 \rho \times m}{r} = \frac{4\pi G r^2 \rho m}{3}$$

So gravitational potential energy (GPE) of X is: $GPE = \frac{4\pi G r^2 \rho m}{3}$

The kinetic energy (KE) of X is given by: $KE = \frac{1}{2} m v^2$

But, as we know from Hubble Law: $v = Hr$

So: $KE = \frac{1}{2} m H^2 r^2$

- If $GPE < KE$ galaxy X continues to travelling forever.
- If $GPE > KE$ galaxy X will eventually stop and be pulled back.

Critical Density (ρ_c) of the Universe

The limiting condition is when $GPE = KE$ and then:

$$\frac{4\pi G r^2 \rho_c m}{3} = \frac{mH^2 r^2}{2}$$

From which the critical density (ρ_c) of the universe is given by:

$$\rho_c = \frac{3H^2}{8\pi G}$$

As you can see, critical density is essentially a collection of constants, thus you can calculate its value as:

$$\rho_c = \frac{3H^2}{8\pi G} = \frac{3 \times (2.3 \times 10^{-18})^2}{8\pi \times 6.67 \times 10^{-11}} = 9.5 \times 10^{-27} kgm^{-3}$$

Using the cosmological principle that the universe is homogeneous, the mean density, ρ, of the universe is found to be $\approx 10^{-27} kgm^{-3}$.

Therefore:

$$\rho_c = 9.5 \times 10^{-27}\ kgm^{-3}$$
$$\rho \approx 10^{-27}\ kgm^{-3}$$

- If the universe's current density, ρ, is less than the critical density, ρ_c, it will continue to expand forever.
- If the universe's current density, ρ, is greater than the critical density, ρ_c, the universe will eventually stop expanding and collapse, resulting in the big crunch.

You can see that ρ and ρ_c are of the same order of magnitude, but because neither is known accurately, it is currently impossible to tell which is larger and hence predict the universe's fate. The majority of cosmologists believe that $\rho = \rho_c$ and advocate for a flat universe. This is where the density is high enough to prevent the Universe from expanding forever but not so high as to trigger a collapse.

The Universe

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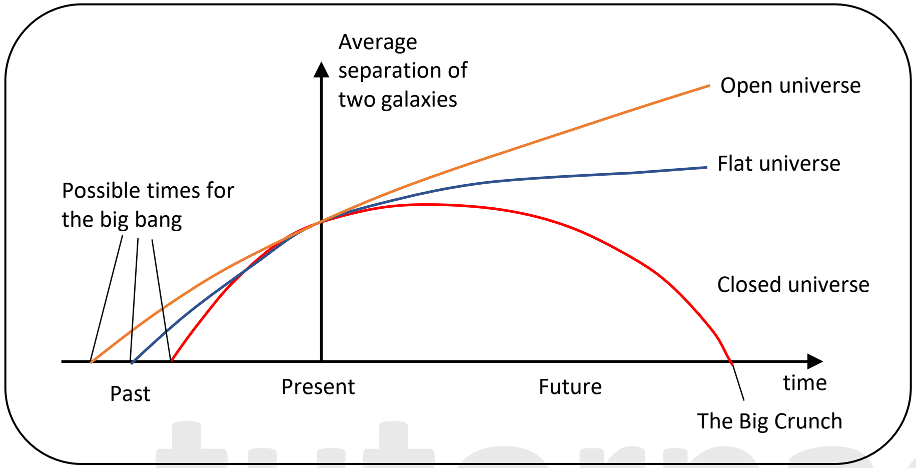
Possible Fates of the Universe

The diagram below shows the three possible scenarios for the future of the universe.



- **Open Universe theory:** Density of universe < critical density

Theory suggests that the universe will expand forever.

- **Flat Universe Theory:** Density of universe = critical density

Theory suggests that the universe will expand towards a finite limit

- **Closed Universe Theory:** Density of universe > critical density

Theory suggests the universe will eventually stop expanding and then contract leading to the big crunch.

It is difficult to predict the future of the universe because of the existence of dark matter, blackholes, neutrinos, dark energy and also because Hubble's constant is not known accurately.

Also keep in mind that each possibility suggests a different time that the big bang occurred. When the precise timing of the big bang is determined in the future, we will be able to predict the universe's fate with some accuracy.

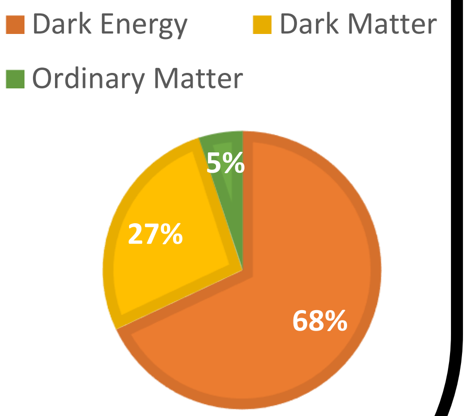
Dark Energy and Dark Matter.

Ordinary matter that we can see and feel makes up only 5% of our universe. As a result, we only understand 5% of the universe.

We know the Universe is expanding at a faster rate, as evidenced by the Hubble Space Telescope, which is used to observes distant supernovae and shows that they are actually speeding up. So the Universe is expanding at an increasing rate, and the cause of this cosmic acceleration is unknown, but it has been called "**dark energy**." Dark energy is a theorised form of energy that fills the entire universe and accelerates its expansion. It is used to explain the rapid expansion of the universe, and it should account for two-thirds of all that exists, but we have no understanding what it is or how to observe it.

Observations of galaxies revealed that stars in galaxies rotate at similar speeds regardless of their distance from the galactic centre. This is in direct contrast to how planets behave in our Solar System, where planets farther from the Sun orbit more slowly than planets closer to the Sun, and this is the case in smaller mass systems. So we know that galaxies rotate, which means that all of the stars in them must be subjected to a centripetal force toward the galaxy's centre of rotation.

When astronomers measure the rotational speed of the galaxies' stars, they find that the calculated mass is insufficient to generate the centripetal force required to keep the galaxy spinning. In fact, the mass of stars is usually just about 10% of what is required. This implies that galaxies must contain a significant amount of mass that does not emit light. This unknown, invisible mass is known as **dark matter**. Astronomers have not yet figured out what dark matter is. Dark matter should account for 27% of the universe's mass. We know very little about it because it can't be observed with telescopes and doesn't interact with light.



Please see ***'12.3.2 Cosmology worked examples'*** **pack for exam style questions.**

For more revision notes, tutorials and worked examples please visit www.tutorpacks.co.uk.